Annual Reviews in Control 42 (2016) 271–284

Contents lists available at ScienceDirect

Annual Reviews in Control

journal homepage: www.elsevier.com/locate/arcontrol

Review

Dynamics and control technologies in air traffic management

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ARTICLE INFO

Article history:
Received 29 August 2016
Accepted 19 September 2016
Available online 29 September 2016

Keywords:
Air traffic management
Global satellite navigation system
Air traffic throughput
Trajectory optimization
Optimal control theory
Model predictive control
Discrete event systems
Trajectory-based operations

ABSTRACT

Dynamics and Control technologies play a central role in the development and operation of decision support systems of modern air traffic management systems. Recent emergence of Global Navigation Satellite Systems and satellite-based augmentation systems have enabled higher precision execution of aircraft trajectories, opening-up the potential for the implementing more quantitative air traffic management approaches. Already, this navigation capability is enabling higher traffic through puts, and safer operation of aircraft in the proximity of the terrain at several major airports in the US. This paper discusses the aircraft trajectory optimization, conflict resolution algorithms, and traffic flow management problems which form the essential components of the evolving air traffic management system. It will be shown that Optimal Control Theory, Model Predictive Control and the Discrete Event Systems theory form the underlying analytical machinery in this domain. Finally, the paper will outline some of the algorithms for realizing the Trajectory Based Operations concept, currently being developed for future air traffic management.

1. Introduction

Air traffic volume has been steadily increasing over the past 4 decades, accelerated by the worldwide deregulation of the industry in the 1980′s. According to the IATA (www.iata.org), nearly 3 billion passengers and over 50 million metric tons of cargo were transported by air in 2013. During that year, Aviation supported 57 million jobs and generated over US$2.2 trillion in economic activity, worldwide. By some estimates, world aviation is expected to grow by 25–30% in the next decade. The accompanying increase in the number of aircraft utilizing the air transportation resources will require substantial modifications to the present air traffic control configurations and procedures. Even if the air transportation safety metrics manage to remain at the present levels, this large increase in traffic volume will adversely impact the system throughput. In anticipation of this fact, Federal Aviation Administration (FAA) in the United States (US) and the EUROCONTROL Organization have initiated the NextGen and the SESAR programs, respectively. The objective of these efforts is to facilitate a safe path to scaling the air traffic control system without compromising performance. In view of the sweeping changes that are required to enable this transition, the system has been renamed as the Air Traffic Management System in recent years.

The objectives of the next-generation air traffic management systems are to transform the system from a largely reactive system to one that employs predictive operations. This will involve automating some of the system functions, and developing decision support systems for others. The NextGen system is expected to eliminate wasteful surface and airborne procedures such as holds at taxiways and runway thresholds, allow continuous climb to cruise, eliminate airborne traffic flow metering or holds, and continuous descend arrivals. Moreover, it is expected that the emerging system will allow for more collaborative traffic flow decisions, involving all the stakeholders.

It is generally agreed that the initial impetus for the development of modern radar-based air traffic control technology began in the US with a series of highly publicized accidents, the first one being a mid-air collision over the Grand Canyon at 21,000 feet altitude, on June 30, 1956 at around 10:30 am Pacific Standard Time, between a United Airlines Douglas DC-7 and Trans World Airlines Lockheed L-1049 Super Constellation. The main impetus for air traffic control system developed was to meet the aircraft conflict detection/resolution objective.

As the traffic volume started increasing in the 70′s, the demands on the available airspace and airport capacities in the vicinity of major population centers during peak traffic hours were often exceeding capacity. The demand-capacity mismatch became even more acute in the presence of adverse weather conditions. Traffic flow management initiatives attempt to address this demand-capacity mismatch.

Currently, the air traffic management system is human-centered, in which the controllers monitor the air traffic through radar-transponder based surveillance and VHF/UHF radio communications with the pilots to ensure conformance with filed flight

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http://dx.doi.org/10.1016/j.arcontrol.2016.09.012
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plans and approve any changes to them, while ensuring the separation between aircraft. The airspace segmented into the air traffic control Centers and Sectors, with individual Sector Controllers ensuring aircraft separation while aligning traffic flow objectives with Center-level traffic coordinators. The terminal areas may similarly be segmented into Sectors. Such an approach breaks the traffic control problem down into a series scalar flow and separation control problems, amenable to manual control requiring virtually no automation.

As the traffic volume increases, the air traffic management requires the coordination of more complex simultaneous interactions between multiple traffic streams to ensure conflict free merging and spacing to ensure efficient traffic flow. Purely manual control is not being practical without substantially increasing the number of sectors with the attendant communication and coordination difficulties. Moreover, since the traffic flow management is based on predictions, decision support tools based on sound analytical algorithms are essential for implementation.

Recent availability of widespread Global Navigation Satellite Systems (GNSS) and satellite-based augmentation systems such as the WAAS in the USA, EGNOS in Europe, MSAS in Japan, and GAGAN India, together with the emergence of wireless data communication technologies have provided the basis for substantial increase in the precision of executing aircraft trajectories. In addition to allowing more precise management by human controllers, these technologies offer the potential for automating several of the lower-level controller tasks, elevating the human controllers to role of traffic managers. Just as automatic flight control technologies have enabled substantial reduction in pilot's cockpit workload, emerging automation tools are expected to reduce controller workload and enhance throughput. High-level decisions may continue to be under manual control, with more routine activities such as separation assurance being handled by automation both on ground and onboard aircraft. By reducing the potential for human error, such automation tools may enhance the overall system safety.

The parallels between flight controls and modern air traffic management is striking. In the flight control arena, the cockpit automation began in 1912 with a two-axis autopilot developed and demonstrated by Lawrence Sperry(McRuer, Ashkenas, & Graham, 1973). It was then followed by the development of stability augmentation systems for emerging large and high-performance aircraft. Altitude hold autopilots appeared during the latter part of World War II. The cold war produced rapid advances in the flight control technologies, culminating in the availability of the first fly-by-wire airliner in the 1970′s with an onboard flight management system. Flight control technology has now reached a highly advanced state with the full authority fly-by-wire digital flight control systems being standard equipment on modern-day airliners. In these aircraft, the pilot's role is largely that of a flight manager responsible for selecting the modes and commands to be executed by the flight control system. The pilot is expected to intervene only if the automation is unable to deal with the situation at hand. On some of the more advanced large aircraft, it is possible to auto-taxi to the runway, takeoff, cruise and land automatically, with moderate degree of pilot interaction.

Automation in air traffic management appears to be following a similar developmental pathway. Research over the past three decades have been focused on developing decision support systems for the controller, wherein the automation synthesizes advisories based on the sensor data, which the human controller then decides to either discard or implement. Algorithms from the Systems and Control discipline are at the heart of these advisory systems. Techniques such as model-predictive and optimal control, linear and nonlinear programming algorithms, dynamic programming and advanced state estimation techniques are all being employed in these algorithms.

As user experience is being accumulated with this approach to graduated automation, down the road, it is likely that human controllers will be relieved of some of the lower level tactical functions such as separation assurance and en route flow control, allowing them to focus on more strategic air traffic management objectives. The air traffic automation system will then form the "outer loop" around the flight control systems onboard individual aircraft to automatically meet most of the tactical air traffic management objectives, with minimal supervision from human controllers.

The emergence of low-cost unmanned aircraft systems (UAS) is accelerating the trend towards automation, due to the sheer number of aircraft that will soon the airspace, both at low altitudes and higher altitudes. This fact has prompted some industry experts to speculate that it is higher likely that extensive air traffic management automation may occur sooner than anticipated. Systems and Control technologies will be central to this transition.

2. Airspace organization and air traffic management

Air traffic management techniques discussed in this paper applies to controlled airspace governed by the Instrument Flight Rules (Federal Aviation Administration, 2016), covering the Class A en route airspace between 18,000 and 60,000 feet Class E transition airspace between 10,000 and 18,000 feet, and the lower altitude Class B regions around major airports. Flight operations in airspace categories such as Class C, Class D, and Class G are covered under different sets of regulations. Every aircraft operating within Class A and B airspace are required to file flight plans with the FAA, and must have approved flight plans before undertaking their operations.

Flight plans generally specify proposed departure time, cruise altitudes, key waypoints along the route, and the destination airport. The expected time of arrival may also be specified in some cases. Air traffic managers analyze the flight plans relative to the traffic demand, and approves or rejects the flight plans. In some cases, amendments may be requested to ensure compliance. Approved flight plans are executable without delays under prevalent weather conditions. However, unexpected weather phenomenon such as storm fronts and other dynamic weather can cause airborne aircraft to request deviations from their original flight plans, potentially causing demand-capacity imbalances, especially near population centers. Air Traffic Management attempts to ameliorate these imbalances while maintaining the FAA-mandated separation between aircraft. Specific responsibilities of the ATM are:

1. Prevent conflicts and ensure adequate separation between aircraft (for maneuvering and wake vortex avoidance)
2. Meet traffic flow control objectives such as matching the demands with available capacities, maximizing throughput and minimizing delays under normal and abnormal operations.
3. Enable access to favorable weather (Tailwinds in Cruise, Headwinds and small Crosswinds for takeoff and landing) and help navigate around unfavorable weather (Icing and Convective Weather)
4. Facilitate navigation around restricted/special use and military airspace, and aviation hazards on the ground
5. Facilitate minimal-delay departures, arrivals, and taxi to and from gates
6. Promote operational procedures for noise abatement and minimizing emissions to minimize the environmental impact of aviation.

In the United States, the FAA has the responsibility to ensure that aircraft operators accessing the national airspace adhere to all the Federal Aviation Regulations. Some of the areas where dynamics and control technologies that impact various aspects of

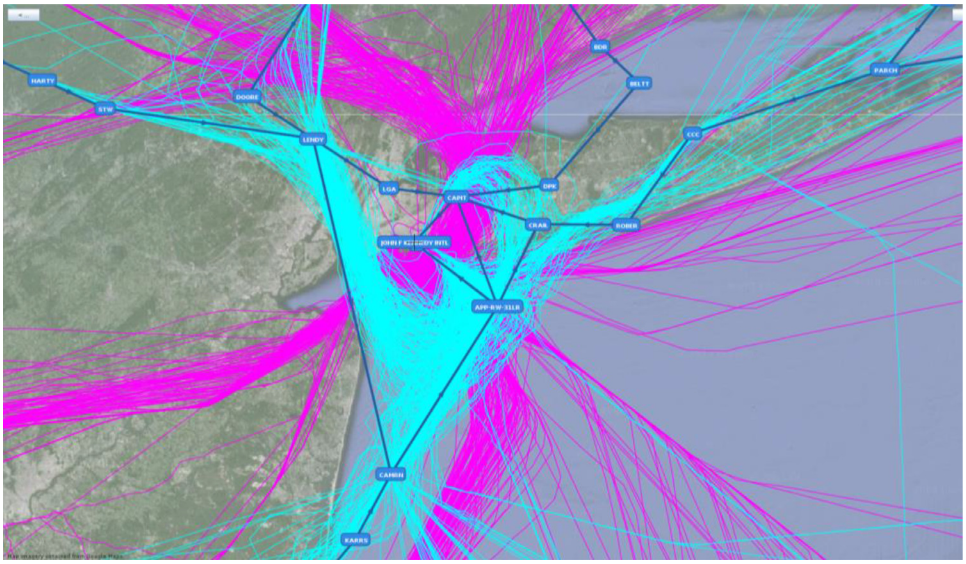

Fig. 1. Simulated tracks of arriving and departing flights at JFK.

the air traffic management problem are discussed in the following sections.

3. Dynamics and control in air traffic management

Aircraft is the fundamental unit in the air transportation system. Consequently, optimal operation of individual aircraft is crucial to the overall system efficiency. Aircraft operators employ optimal trajectories to minimize fuel consumption, while meeting published schedules. Ideally, every aircraft would be allowed to operate along their optimal climb-cruise-descent trajectories, taking advantage of winds where feasible, and avoiding adverse weather as they are encountered. However, since the airspace resources are shared by multiple aircraft, execution of individual aircraft flight plans will be constrained by the presence of other aircraft operating in the airspace.

The objective of the ATM is to enable relatively unimpeded operation of all aircraft in the airspace, intervening only when the aircraft deviate from their flight plans, or if there is a potential for conflict. Although over 6000 aircraft operating in the US national airspace system at any time instant occupy a negligible amount of airspace, difficulties emerge because arriving air traffic is converging and descending towards airports, while departing traffic may climb through the same routes. As an example, simulated traffic flow on a typical day in the vicinity of New York's JFK airport is given in Fig. 1. Additional difficulties arise because fixed-wing aircraft have limited ability to speed up or slow down, and can only fly a limited amount of time due to the limited amount of fuel they can carry. Finally, landing/takeoff operations on runways are limited to around 30 aircraft/hour on each available runway.

These factors and constraints give rise to rich variety of algorithmic challenges some of which are discussed in the following sections. However, it is to be emphasized that the material presented in this paper is hardly exhaustive, since the NextGen program is evolving and a large numbers papers continue to be published at various international conferences and journal on the related problems.

3.1. Aircraft trajectory optimization

Energy efficient operation of the air transportation system is in everyone's interest. In addition to reducing the operating costs for airlines, energy efficient operations will decrease the environmental impact of aviation operations. However, on-time arrival at destinations is a measure of the airline operational success, and a metric by which a large segment of the travelling public judges the airline performance, the "Direct Operating Cost" (DOC) criterion has been adopted as the operational metric by most airlines (Erzberger & Lee, 1980). Flight plans are designed for submission to the FAA for approval based on this criterion, as well as for implementation on onboard Flight Management System (FMS). The DOC criterion is a weighted linear combination of the fuel expended and flight time to transit from the origin to the destination. This criterion has the form:

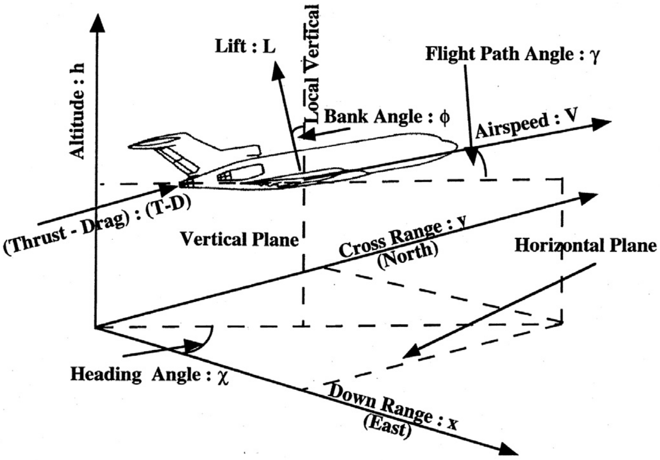

Fig. 2. Coordinate system defining the point mass model.

$$DOC = Fuel + CI \cdot Time \tag{1}$$

The relative weight *CI* between fuel and flight time, which is cost index, is chosen by the airline for individual aircraft, based on the fleet operations criteria. Ambient winds and en route dynamic weather are included as constraints in the trajectory optimization process. Point-mass aircraft dynamics (Fig. 2) defines the differential constraints on the optimal control problem imposed by the aircraft dynamics. These are given in the form (Kelley, 1973), (Park & Clarke, 2012):

$$\dot{V} = \frac{\eta T - D}{m} - g\sin\gamma - \cos\gamma\left(\dot{U}_w \cos\chi + \dot{V}_w \sin\chi\right) \tag{2}$$

$$\dot{\gamma} = \frac{L\cos\phi}{mV} - \frac{1}{V}g\cos\gamma + \frac{\sin\gamma}{V}\left(\dot{U}_w\cos\chi + \dot{V}_w\sin\chi\right) \tag{3}$$

$$\begin{aligned} \dot{\chi} &= \frac{L\sin\phi}{mV\cos\gamma} + \frac{1}{V\cos\gamma}\left(\dot{U}_w\sin\chi - \dot{V}_w\cos\chi\right) \\ \dot{m} &= -\eta Q \end{aligned} \tag{4}$$

$$\dot{x} = V\cos\gamma\cos\chi + U_w \tag{5}$$

$$\dot{y} = V\cos\gamma\sin\chi + V_w \tag{6}$$

$$\dot{h} = V\sin\gamma \tag{7}$$

In these equations, V is the airspeed, γ is the flight path angle, χ is the heading angle, h is the altitude, x is the down range and y is the cross range. U_W and V_W are the wind velocity components along the downrange and crossrange directions. The control variables in the point mass model are the lift L, the bank angle ϕ and the throttle η. T is the maximum engine thrust, D is the drag and g is the acceleration due to gravity. Initial and final values of the down range, cross range and altitude are generally specified in aircraft trajectory optimization problems. The flight dynamics of the aircraft is constrained by lift, throttle and bank angle limits. Moreover, the minimum and maximum airspeeds and the climb rate at different altitudes are constrained, as is the maximum altitude.

The problem of determining the trajectories that minimize the performance index subject to differential constraints (2) through (7) forms an Optimal Control problem (Burrows, 1983; Kelley, 1973; Menon, Kelley, & Cliff, 1985; Sorensen & Waters, 1981; Vaddi, Sweriduk, & Tandale, 2012). Numerical trajectory optimization techniques such as collocation (Garg et al., 2010; Ross & Fahroo, 2004) and multiple shooting (Bock & Plitt, 1984) can be used to synthesize optimal trajectories. However, due to the extreme numerical sensitivity of the costates, the solution process is far from routine, and the convergence is not always guaranteed.

Intense research in this area during 1960–1990 has produced set of highly accurate reduced-order models (Kelley, 1973) that can be used to derive near-optimal results. These approximations are derived by assuming that the $L \cong mg$. Additionally, if the altitude is considered to be a “control like” variable (Kelley, 1973), (Kelley, Cliff, & Weston, 1983) one can derive the “Energy State Model”.

$$\begin{aligned} \dot{E} &= \frac{(\eta T - D)V}{mg} \\ \dot{\chi} &= \frac{g\sin\phi}{V} + \frac{1}{V}\left(\dot{U}_w\sin\chi - \dot{V}_w\cos\chi\right) \\ \dot{m} &= -\eta Q \\ \dot{x} &= V\cos\chi + U_w \\ \dot{y} &= V\sin\chi + V_w \end{aligned} \tag{8}$$

where $E = h + V^2/2g$.

In this model, the control variables are the altitude, heading angle and throttle. A model intermediate in complexity between the point mass model and the energy state model can also be derived. This model also assumes $L \cong mg$. However, it employs the flight path angle as the control variable in the vertical plane.

Solutions to various aircraft trajectory optimization problems using both these models have been reported in the literature (Erzberger & Lee, 1980; Kelley, 1973; Menon et al., 1985). The approach first decomposes the aircraft trajectory into climb, cruise and descend phases. Near-optimal trajectories in each phase can be computed through an algebraic optimization process satisfying the following conditions.

$$\text{Climb}\begin{cases} \left.\dfrac{d}{dh}(V(T-D))\right|_{E=const} = 0 & (Time\ Optimal) \\ \left.\dfrac{d}{dh}\left(\frac{V(T-D)}{Q}\right)\right|_{E=const} = 0 & (Fuel\ Optimal) \end{cases} \tag{9}$$

$$\text{Cruise}\quad \frac{\partial}{\partial V}\left[\frac{QD}{VT}\right] = 0, \quad \frac{\partial}{\partial h}\left[\frac{QD}{VT}\right] = 0 \tag{10}$$

$$\text{Descent}\quad \left.\frac{d}{dh}(T_{idle} - D)\right|_{E=const} = 0 (Maximum\ Range) \tag{11}$$

where $\frac{d}{dh}(\cdot)|_{E=const} = \frac{\partial}{\partial h}(\cdot) - \frac{g}{V}\frac{\partial}{\partial V}(\cdot)$; T and Q are maximum thrust and associated fuel flow rate; T_{idle} is the idle thrust.

These trajectory computations are non-iterative, and are simple enough to be implemented onboard the FMS. Indeed, several FMS models available on modern airliners use the energy state models to synthesize flight trajectories.

Several variations of these results have been derived in the literature considering current traffic procedures and operational concepts: energy and time optimal speed profile with fixed path for the landing trajectory generation (Zhao & Tsiotras, 2013a, 2013b), and DOC and fuel optimal solutions with Required Time-of-Arrival (RTA) including latter portion of cruise segment for the arrival phase (Park, 2014; Park & Clarke, 2016).

The data required for the computation of optimal trajectories of over 400 modern commercial aircraft are available in Base of Aircraft Data (BADA) (Nuic, 2014). As an example, Fig. 3 shows the examples of DOC optimal descent trajectories for B737-500 and B767-400. BADA is used as an aircraft performance model (Nuic, 2014). The lateral path is fixed according to the SADDE 6 Standard Terminal Arrival Route (STAR) into the Los Angeles International airport.

3.2. Conflict resolution algorithms

As indicated in the previous section, operations with optimized trajectories are essential for efficient operation of aircraft. However, since there are other aircraft with the same objectives operating in the vicinity, aircraft can sometimes be in conflict with each other. Conflicts can occur with higher frequencies in busy terminal areas, although they may also arise en route when aircraft climb/descend or cross other aircraft. As an example, a schematic of the arrival and departure traffic routes to and from the San Francisco bay area airports, given in Fig. 4. The three major airports in this region are the San Francisco (SFO), Oakland (OAK) and San Jose (SJC) international airports.

Currently, one of main responsibilities of air traffic controllers is that of resolving conflicts between aircraft en route and in the terminal areas. In the US, the federal aviation regulations require that en route aircraft be separated by a minimum of 5 nautical miles while operating at the same altitude, or be separated by a minimum of 3 nautical miles while in the terminal area. The minimum altitude separation between aircraft is required to 1000 feet. Along busy airways, the FAA has established reduced vertical separation standards (RVSM) permitting vertical separation between aircraft to 500 feet. Aircraft that approach closer than the specified separation standards are considered to be in conflict. A notional definition of aircraft conflicts in the horizontal plane is given in Fig. 5.

Whenever the potential for a conflict is detected, the air traffic controllers issue advisories consisting of speed, heading and altitude changes, in that order of preference, to resolve any impending conflict. Conflict resolution is a high workload activity, and is potentially one of the air traffic management tasks that are likely to handled by automation onboard aircraft in the future.

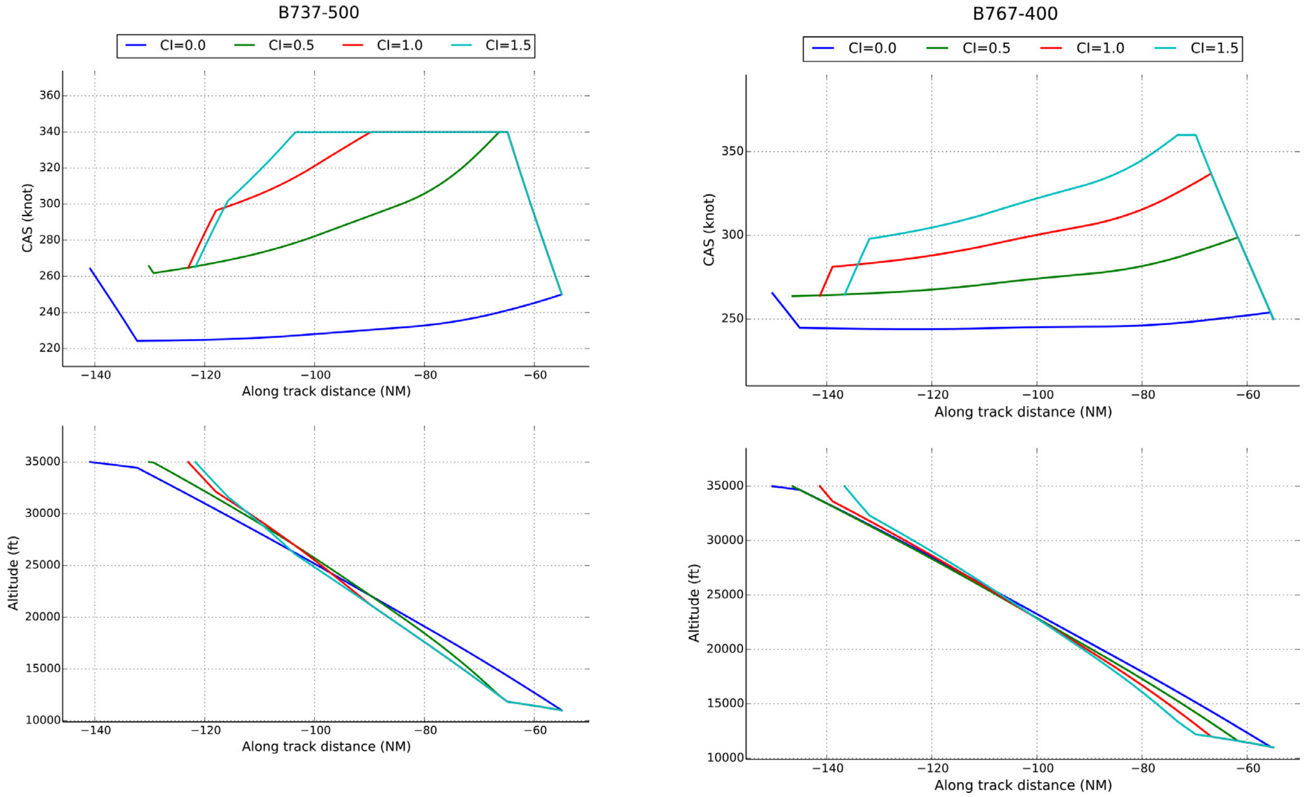

Fig. 3. Minimum DOC descent trajectories with various CIs.

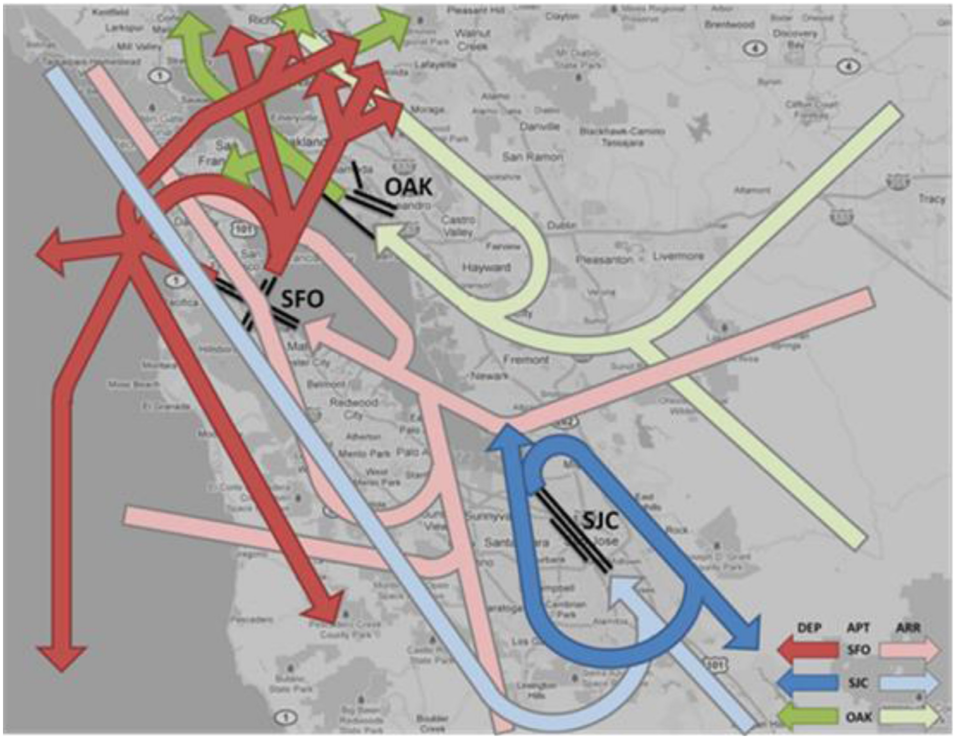

Fig. 4. Traffic routes for approaching and departing traffic at the San Francisco Bay area airports.

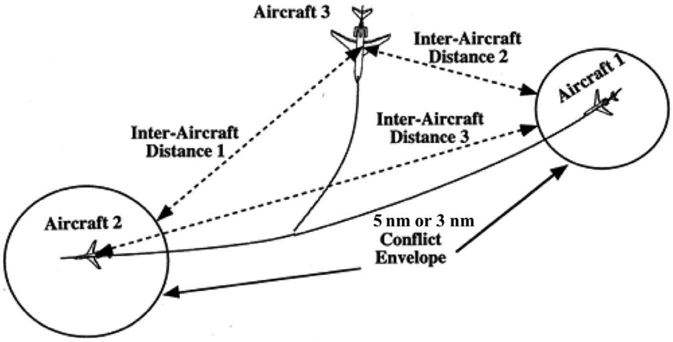

Fig. 5. Notional definition of aircraft conflicts.

If there are n aircraft involved in a conflict, the conflict resolution process will have to be address $n(n-1)/2$ relative distances that must satisfy the separation standards. If $n=2$, the process is relatively straightforward. However, if more than two aircraft are involved, as may happen in the future when the traffic densities go up, methods that explicitly address the simultaneous conflict resolution problem must be addressed. Since the conflict resolution process involves tactical maneuvers, these problems are generally formulated using kinematic models of aircraft without the consideration of wind, as follows:

$$\dot{x} = V\cos\gamma\cos\chi\,,\ \dot{y} = V\cos\gamma\sin\chi\,,\ \dot{h} = V\sin\gamma$$

The conflict resolution problem can then be formally stated as one in which every aircraft involved in the conflict attempts to minimize deviations from their nominal trajectories, while ensuring that the inter-aircraft separation constraints are satisfied. A formulation of the conflict resolution problem based on numerical trajectory optimization was advanced in (Menon, 1993; Menon, Sweriduk, & Sridhar, 1999). Piecewise linear parameterization of the trajectories was employed in conjunction with a sequential quadratic programming algorithm to derive optimal conflict resolution commands. Multiple conflict resolution scenarios were illustrated in that work. A six example merging scenario from that work is illustrated in Fig. 6.

The inter-aircraft distance shown on the bottom left hand side of this figure shows that several aircraft were in conflict before that application of the conflict resolution methodology. The bottom right hand side subfigure shows that the algorithm resolved all the conflicts.

Subsequently, the problem of conflict resolution between two aircraft was formulated using a kinematic model of the aircraft (Bilimoria, 2000). Several other numerical optimization-based and heuristic approaches have been advanced in the literature for the conflict resolution problems. References Eby, 1995; Frazzoli, Mao, Oh, and Feron, 2001; Hong, Choi, Lee, and Kim, 2016; Hwang and Tomlin, 2002; Liu and Hwang, 2014; Pallottino, Feron, and Bicchi, 2002; Panyakeow and Mesbahi, 2010; Sislak, Volf, and Pechoucek, 2011; Tomlin, Pappas, and Sastry, 1998; Vela et al., 2010 are some of the representative research efforts.

In addition to conflict resolution by the controllers, every aircraft operating in the Class A airspace incorporate Traffic Collision Avoidance System (TCAS) (FederalFederal Aviation Administration, 2011). This system serves as the back up for controller assisted conflict resolution, and has been standardized. Operational details of the TCAS can be found in the literature.

3.3. Traffic flow management

The national airspace, terminal areas and airports are the resources available in the air transportation system, and have physical limits on their capacities. Airlines and general aviation are competing for this resource. The competition is at its highest during peak business hours such as the morning and the evening hours, and is generally the lowest between midnight and early morning. Operating time slots for individual aircraft or airline operations are approved by the regulatory agencies such as the FAA to ensure that the traffic flow demand matches the available capacity. Under normal operating conditions, this process produces little or no delays. However, disruptive events such as dynamic or inclement weather can cause substantial decrease in capacity, causing mismatches between available capacity and traffic demands.

Traffic flow management initiatives attempt to ameliorate such capacity-demand mismatches by delaying aircraft on the ground, generally termed as Ground Delay Program in the US, or in the air if necessary. Airborne delays are introduced by requiring the aircraft to slow down, or by imposing path-stretching maneuvers. As a last resort, aircraft are placed in holding patterns at specific regions of the airspace. Airborne delays are to be avoided, since they result in fuel consumption often at non-optimal operating conditions.

The traffic flow management problem can be formulated as a capacity maximizing control problem subject to the dynamic constraints imposed individual aircraft dynamics. However, such an approach can become rapidly intractable due to the sheer number of state variables in the problem. For instance, if kinematic models for aircraft are employed, a traffic flow control problem consisting of 6000 aircraft will have a minimum of 18,000 states and control variables. The use of full point mass models would double the size of the problem.

An alternate approach is to employ dynamic models describing the aggregate behavior of the traffic in the airspace. These models have the advantage that their orders are independent of the number of aircraft operating in the airspace. The disadvantage is that the aggregation process does not permit the synthesis of control strategies for individual aircraft. Instead, they create control strategies for groups of aircraft, and methods must be found to disaggregate the controls for use with individual aircraft in the group. Nevertheless, the aggregate models have been found useful in strategic air traffic flow management. Three such aggregate models which the authors have been involved in, will be discussed in the following subsections.

3.3.1. Eulerian air traffic flow models

Following the analogy with Fluid Mechanics (Prandtl & Tietjens, 1957), the formulation of the traffic flow problems in terms of individual aircraft dynamics can be termed as the Lagrangian Approach. An alternate approach, known as the Eulerian flow models has been much more successful in the development of central results in Fluid Mechanics. In this latter approach, instead of analyzing the behavior of individual fluid particles, the system dynamics is formulated in terms of aggregate characteristics of the fluid flow at fixed regions of the space are considered.

In the case of air traffic flow, the dynamics of the characteristics of the airspace such as the traffic density and average speed are modeled in terms of the air traffic flow entering and leaving the airspace. As an example, Fig. 7 shows the latitude-longitude discretization of the airspace, including the details of arrival-departure flows into an airport at one of the spatial regions.

Reference Menon, Sweriduk, and Bilimoria, 2004 showed that the application of the flow conservation principle to such an eight-connected airspace discretization yields a set of linear difference equations in terms of traffic flow inertia parameters $a_{i,j}$ and flow deviation parameters $\beta_{p,q}$. The flow inertia parameters characterize the time spent by aircraft in each discretized spatial element along each of the eight directions, while the flow deviation parameters characterize the fraction of the aircraft in the spatial element that

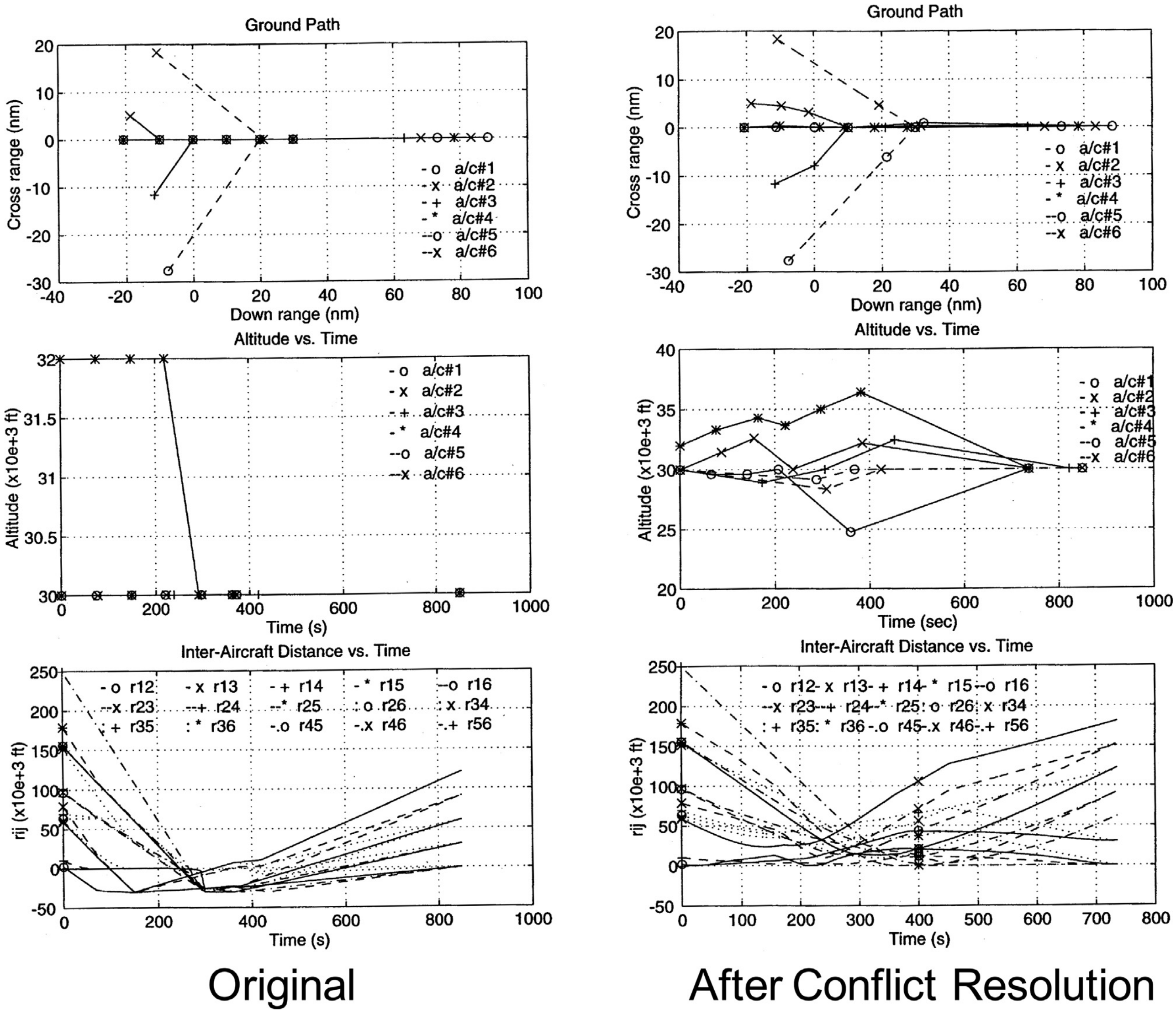

Fig. 6. A Sample air traffic resolution scenario.

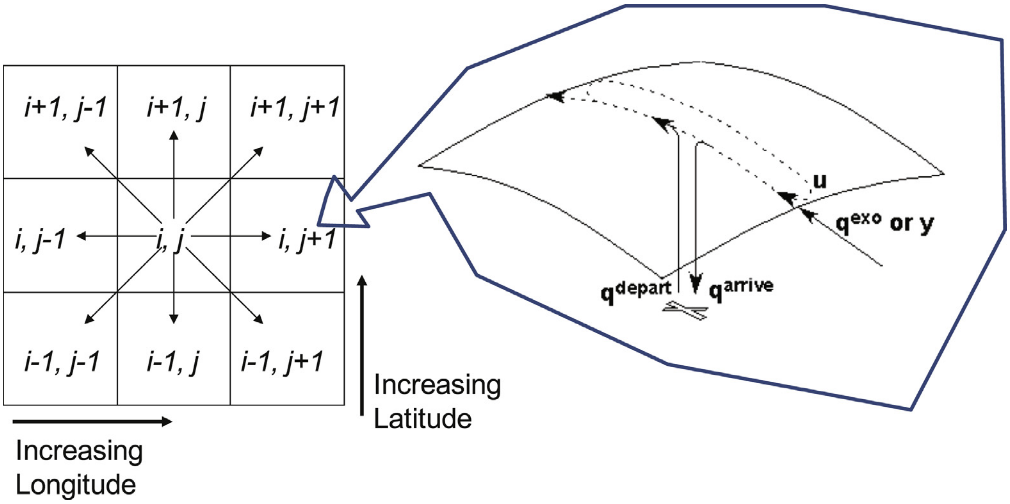

Fig. 7. Eight-connected discretization of the airspace with airport arrival-departures.

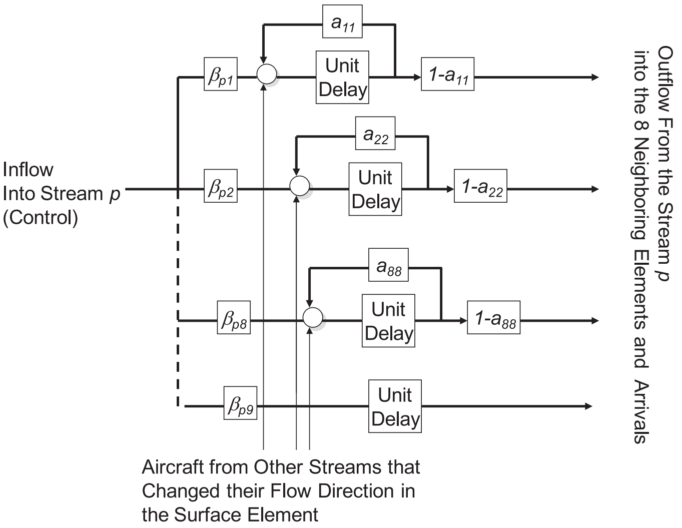

Fig. 8. Dynamics of a spatial element.

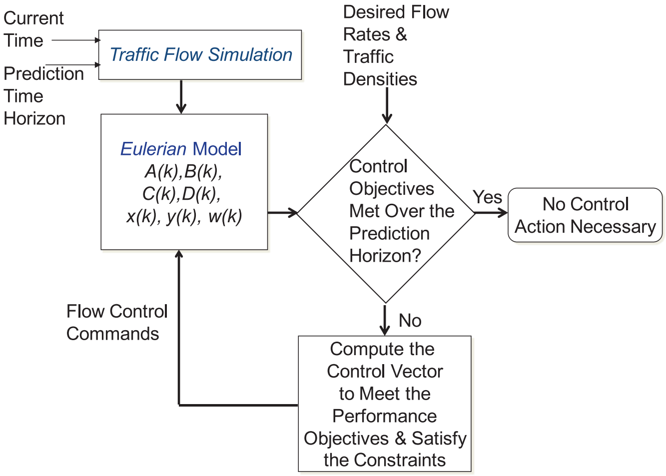

Fig. 9. Model-predictive traffic flow management using eulerian traffic flow model.

changed the flow direction. The state variables in this model are the number of aircraft in a discretized spatial element. It was shown in Menon et al., 2004 that the Eulerian traffic flow model can be represented in the standard state variable form given in the following, and is amenable to analysis using linear algebraic methods.

$$x(k+1) = A(k)x(k) + B_1(k)u(k) + B_2(k)w(k)$$

$$y(k) = C(k)x(k) + D(k)u(k)$$

Moreover, the dynamics of spatial elements can be succinctly represented using block diagrams, as shown in Fig. 8 for a single spatial element. The control variables in this model are the inflow rates of aircraft at various spatial elements. The flow rates indirectly specify the departure delays at the airports, as well as any airborne delays to be imposed by air traffic controllers.

Interestingly, the air traffic flow control based on the Eulerian model parallels the operation of the present day air traffic management system wherein air traffic controllers are assigned one or more Sectors defining a spatial region of the airspace for control, and are tasked with maintaining the air traffic density within specified bounds by adjusting the input/output traffic flow rates. The Eulerian traffic flow model has been used to formulate model-predictive traffic flow management problems (Menon et al., 2004), such as the flowchart of the flow control algorithm is given in Fig. 9. In addition to being useful for formulating flow control problems, the model can be used to analyze the impact of flow uncertainties in one region of the airspace on another region. Under Gaussian uncertainty assumption, the linear Eulerian model readily allows the propagation input covariances, as illustrated in Menon et al., 2004. Other applications of this of the model have been identified designing decentralized flow control strategies, and in the investigation of reachability (Menon et al., 2004).

Several variants of the Eulerian traffic flow models and flow control applications have been discussed in the literature (Sridhar, Soni, Sheth, & Chatterji, 2006; Sun, Sridhar, & Grabbe, 2009). More recently, this model has been used in conjunction with the linear-quadratic regulator theory to derive flow control logic for the terminal area (Bai & Menon, 2015) being some of them.

3.3.2. Traffic flow management using time-of-transit models

The Eulerian traffic flow model casts the flow control problem in terms of linear dynamic models describing the evolution of traffic density in various regions of the discretized airspace. In these models, the number of aircraft in a traffic stream is the dependent variable, and the time is the independent variable. The airspace capacities are specified in terms of the dependent variables.

The time-of -flight models take an alternate point of view by casting the traffic flow control problem as a linear-algebraic optimization problem. In this parallel development, algebraic equations are formulated describing the time of flight as individual aircraft transits through the airspace. The example shown in Fig. 10 illustrates the overall approach. The difference between time of exit and the time of entry in each spatial region defines the time-of-transit through the region. Sum of all the times of transits is the total flight time. Flight delays that may be imposed at specific points in the airspace or at the runways, such as t_{d1} in Fig. 10 can then be used as the optimization parameters.

The flow control problem consist of linear time-of-flight constraints for every aircraft operating in the airspace, with capacity airspace constraints defining the number of aircraft that can simultaneously occupy spatial regions of the airspace. Minimum and maximum times of entry into the airspace regions and minimum and maximum times of exits from the airspace form additional constraints in the problem.

The cost index is the sum of ground and airborne delays for all aircraft under consideration required to meet all the constraints in the problem. Since all the constraints in the problem, as well as the cost index are linear, such a formulation is amenable to solution using the linear programming approach (Bertsimas & Tsitsiklis, 1997). Since the airspace capacity constraints are integer constraints, the problem is a mixed integer-linear programming (MILP) problem. Highly efficient numerical formulation of this MILP problem for air traffic flow control has been advanced in Bertsimas and Patterson, 1998; Bertsimas, Lulli, and Odoni, 2011.

Additional constraints in the traffic flow optimization problem arise from dynamic weather. As an example, some of the jet routes in the US national airspace system, together with weather-constrained airspace are illustrated in Fig. 11. Regions of severe weather are approximated by convex polygons, and the traffic is rerouted around these areas. Thus, the traffic flow optimization problem subject to dynamic weather constraints will have to include not only the nominal aircraft routes, but also the alternate routes to be adopted in case of dynamic weather constraints.

Recently, (Sengupta, Kwan, & Menon, 2015) the linear programming based traffic flow optimization problem has been employed for optimizing the integrated arrival-departure traffic flows. The objective of this research was to ensure an inter-aircraft time separation of 1 minute at the runway, by optimally selecting individual

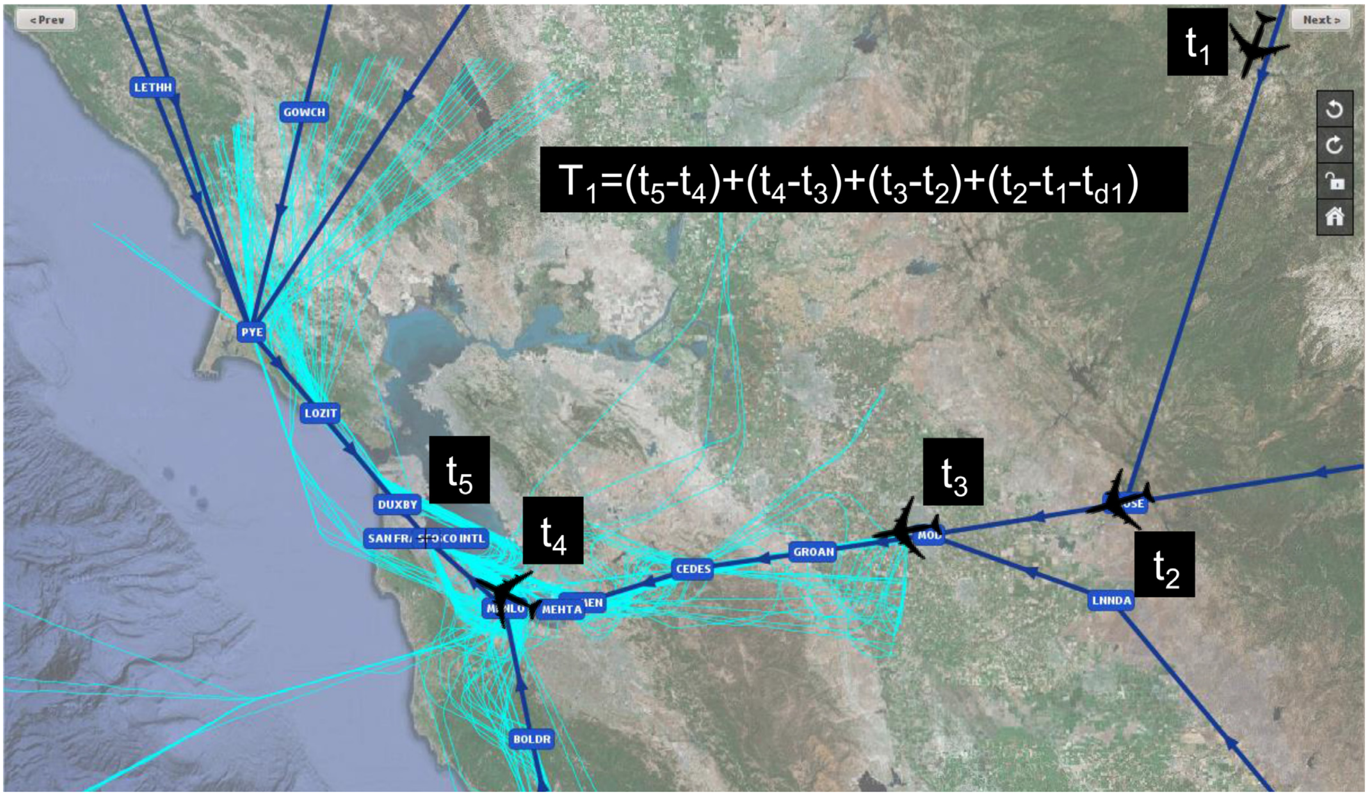

Fig. 10. Linear constraint describing the flight duration, t_{d1} is a control variable.

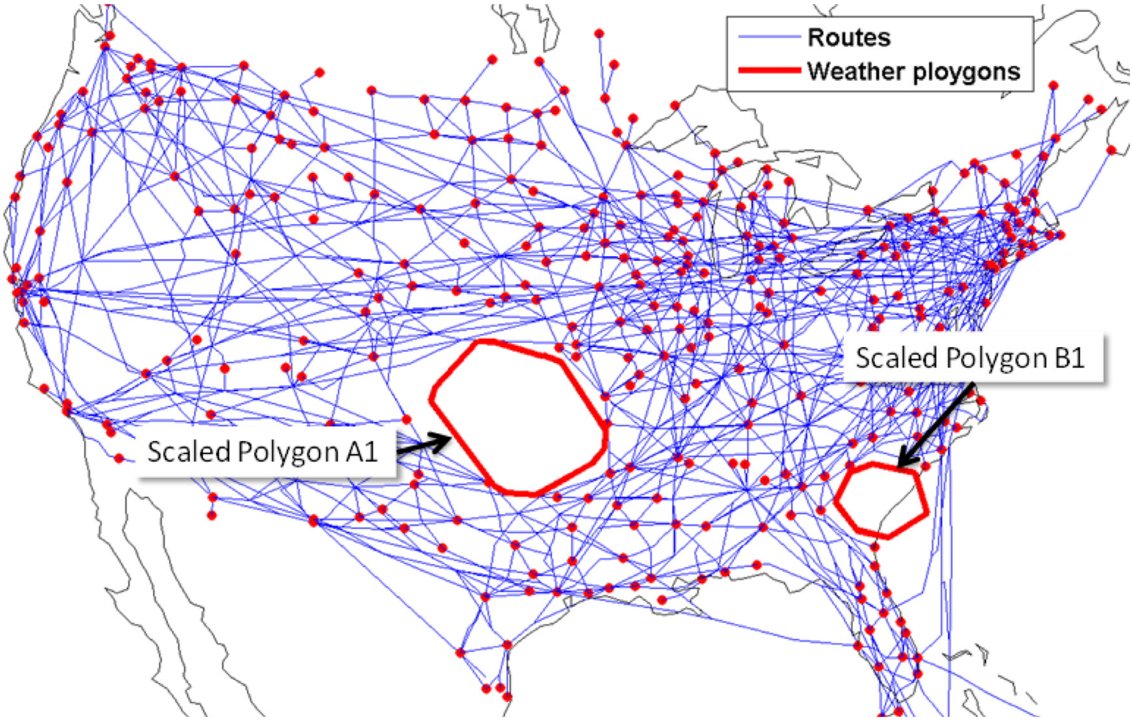

Fig. 11. Reroutes avoiding scaled1 versions of the convex envelope around adverse weather.

aircraft delays over a specified time-window, to minimize the overall traffic flow delay. A sample result from this research is given in Fig. 12.

In order to serve as a basis for assessing the effectiveness of such delay minimization methodology, a simple delay strategy based on a discrete event simulation (DES) was first developed. In this approach, aircraft are delayed sequentially before they arrive at the runway, so as to meet the 1 minute inter-aircraft separation time. Results for an hour of operation is given in Fig. 12. It may be observed that the delays imposed by the optimization algorithm are substantially smaller than those derived from a simple sequential delay process.

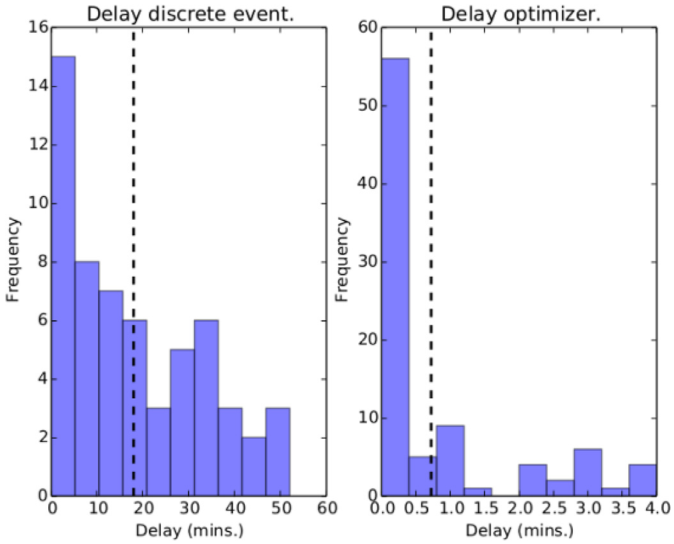

Fig. 12. System delays obtained using DES and optimizer for separation of 60 seconds.

3.3.3. *Queuing network traffic flow models*

Although not strictly a traffic flow control technique, the queuing network models (Hillier & Lieberman, 2001; Cassandras & Lafortune, 2007) have been found to be highly useful in analyzing the characteristics of the traffic flow in the national airspace system. This approach can be conceptualized as a stochastic version of the time-of-flight models, in which, the traffic flow distribution in the national airspace system is assumed to be composed of stochastic traffic flows entering the system, with stochastic flight times between origin-destination pairs. Thus traffic flows in the national airspace system is represented as a series of queues formed by the transient imbalances between arrival flow rate variations and the flight time or the service time variations.

Fig. 13 shows the queuing network abstraction of the national airspace system. Depending on the resolution of the desired analytical results, separate queues can defined for the taxi, takeoff, cruise, descent and landing phases. Moreover, the models can be defined at multiple spatial resolutions. For instance, queuing network model consisting of just the origin-destination nodes can be formulated. The spatial discretization can be at the Center or Sector levels, or on latitude-longitude-altitude grids. Moreover, queuing network models can be constructed for various time periods of the day, as well as for various seasons.

The queuing network abstraction can be used in conjunction with the traffic distribution data at various points in the network can be used to derive highly useful insights to the operation of the system. The starting point for this type of analysis is the derivation of the queuing network parameters from actual traffic data. As an example, Fig. 14 shows the inter-arrival time distribution for 2007 at the Denver International Airport (Tandale et al., 2008). It may be observed that an exponential distribution fits the traffic data very well

Queuing network models of the air traffic flow can be used to derive several useful results. For instance, the impact of various uncertainties on the system performance can be readily assessed using algebraic models. For instance, Fig. 15 illustrates the impact of system uncertainties in taxiway-runway operations and en route wind and weather impact the flight times between several major airports in the US. Another interesting result derived using the queuing network model is given in Fig. 16. This figure illustrates the regions of the airspace where traffic congestions are likely to arise, if the traffic were to increase from the current levels by various the various multiples. Interestingly, the subfigure on the lower right shows that large regions of the airspace will operate at or near capacity if the traffic volume were to double. Queuing networks readily allow the computation of such results using purely algebraic models.

4. Trajectory based operations

Over the past three decades, air traffic management systems have been evolving to meet the rise in demand. A key component in the evolution of the ATM system envisioned through NextGen in the US and the SESAR in the Europe is the Trajectory Based Operations (TBO) concept. The TBO concept is expected to significantly improve the predictability of the traffic by moving away from the current clearance based air traffic control, to the trajectory-based ATC and TFM by considering the whole trajectory from the current state to the final point. By improving predictability through TBO, more efficient traffic management becomes possible, and leading to improvement in capacity of the airspace, especially in terminal areas and the runways. In addition, TBO can reduce environmental impact by maximizing usage of the environmentally friendly operations such as Continuous Descent Arrival (CDA) (Clarke et al., 2004) or Optimized Profile Descent (OPD) (Clarke et al., 2013) in arrival phase, and Optimized Profile Ascent (OPA) in climb phase, which are applicable only when aircraft fly without intervention of the ground controller. Furthermore, by trajectory negotiation, each aircraft can fly along its preferred trajectory, hence TBO can increase the economical benefits of airlines.

The central premise of the TBO is that the aircraft trajectories can be predicted with greater certainty based on the given environmental and traffic information actually experienced by the aircraft. This approach leverages the trajectory prediction capabilities already available on the FMS onboard the aircraft. Aircraft trajectory is partitioned into several segments based on the flight phase and flight mode such as constant Mach/CAS. Trajectories are generated for each flight segment using the point mass model (Slattery & Zhao, 1997) and assembled to create the predicted trajectory. According to the TBO concept, every aircraft in the airspace region under consideration provide these trajectories to the air traffic managers for traffic flow management and separation assurance.

The common requirement in both NextGen and SESAR TBO concepts is the satisfaction of Time-of-Arrival (TOA) constraints. Aircraft operating under TBO will be required to have FMS that can provide TOA control (TOAC) capability consisting of two components: 4D trajectory (4DT) generation capability that satisfies RTA and 4D guidance capability to achieve assigned RTA against uncertainties such as wind forecast error.

Based on the assigned TOA and the TBO concept, there exists several variants for TOAC. Fig. 17 shows a common block diagram of TOAC. The 4D guidance capability implies that the TOAC is integrated with the lateral navigation (LNAV)/vertical navigation(VNAV) features (3D path guidance) features of the FMS. As shown in the figure, the block diagram can be divided into two computational loops: i) trajectory re-planning loop, and ii) feedback guidance loop.

The trajectory re-planning loop is outer loop that synthesizes a 4DT that satisfies the given time constraints at specified waypoints, which is a reference trajectory for the 4D guidance loop. Optimal control based approach (Park, 2014), CI iteration method (DeJonge, 1988), and trajectory sensitivity based method (Jackson & O'Laughlin, 2007; Jackson, Zhao, & Slattery, 1999) have been developed for the trajectory re-planning loop.

The feedback guidance loop, which consists of the components ii the red dash box in Fig. 17, includes LNAV/VNAV for 3D path tracking and a speed adjustment control law for along-path time control.

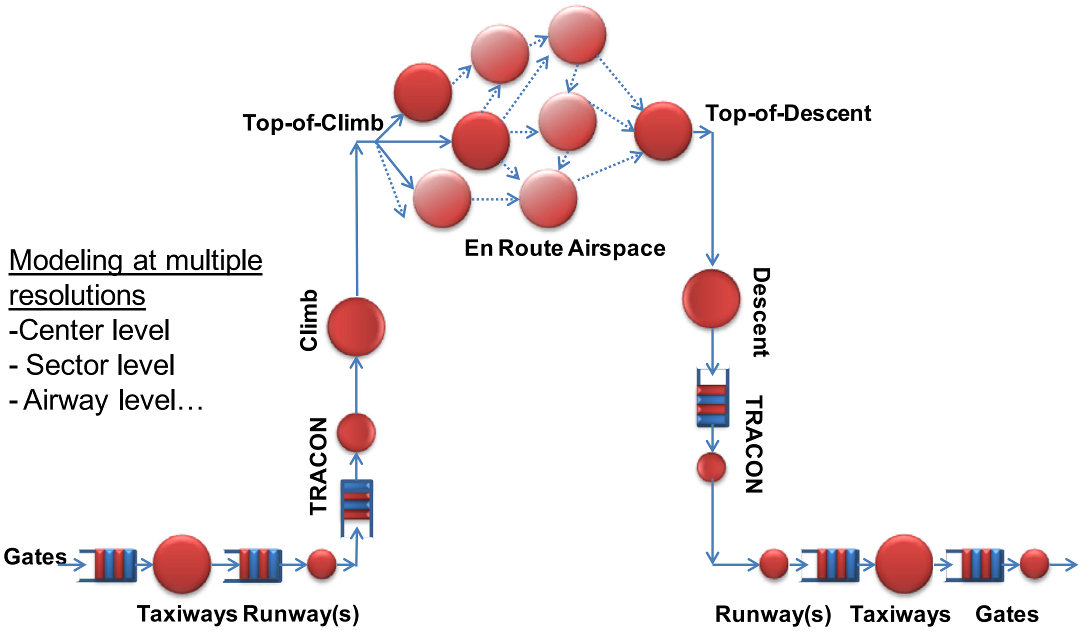

Fig. 13. Queuing network abstraction of the airspace system.

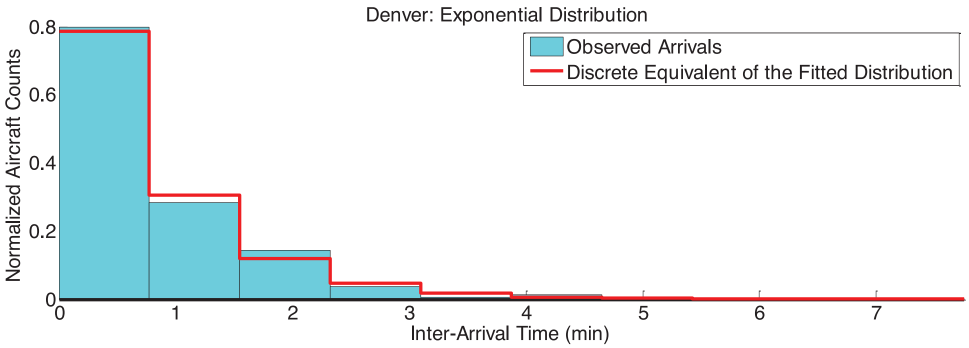

Fig. 14. Inter-arrival time distribution at Denver international airport.

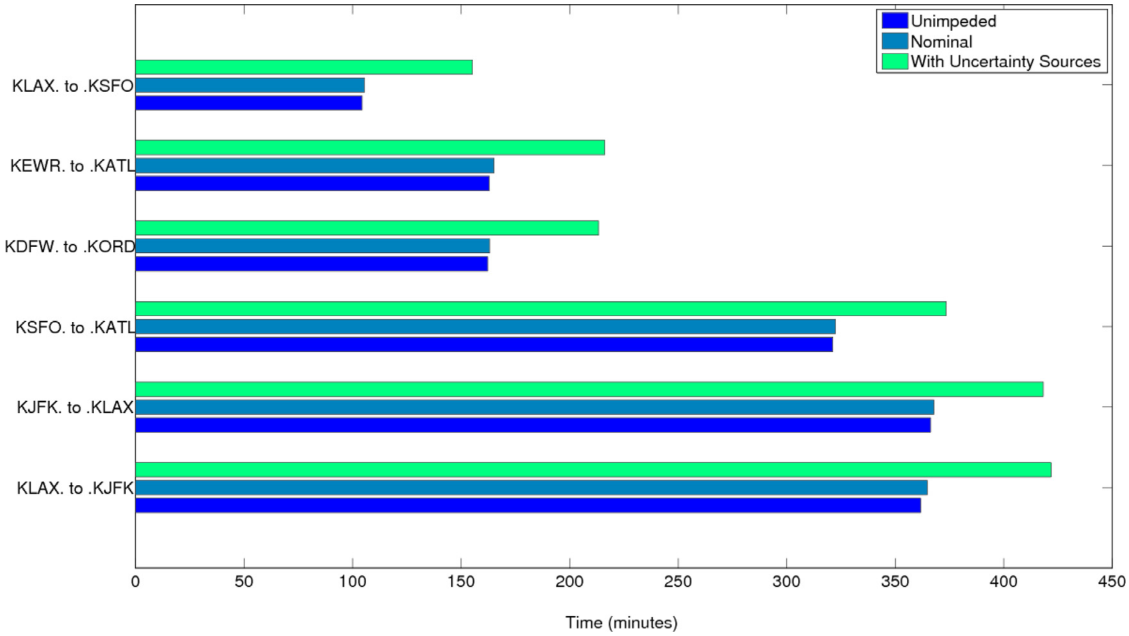

Fig. 15. Mean gate-to-gate delays between several major US airports and Los Angeles international airport.

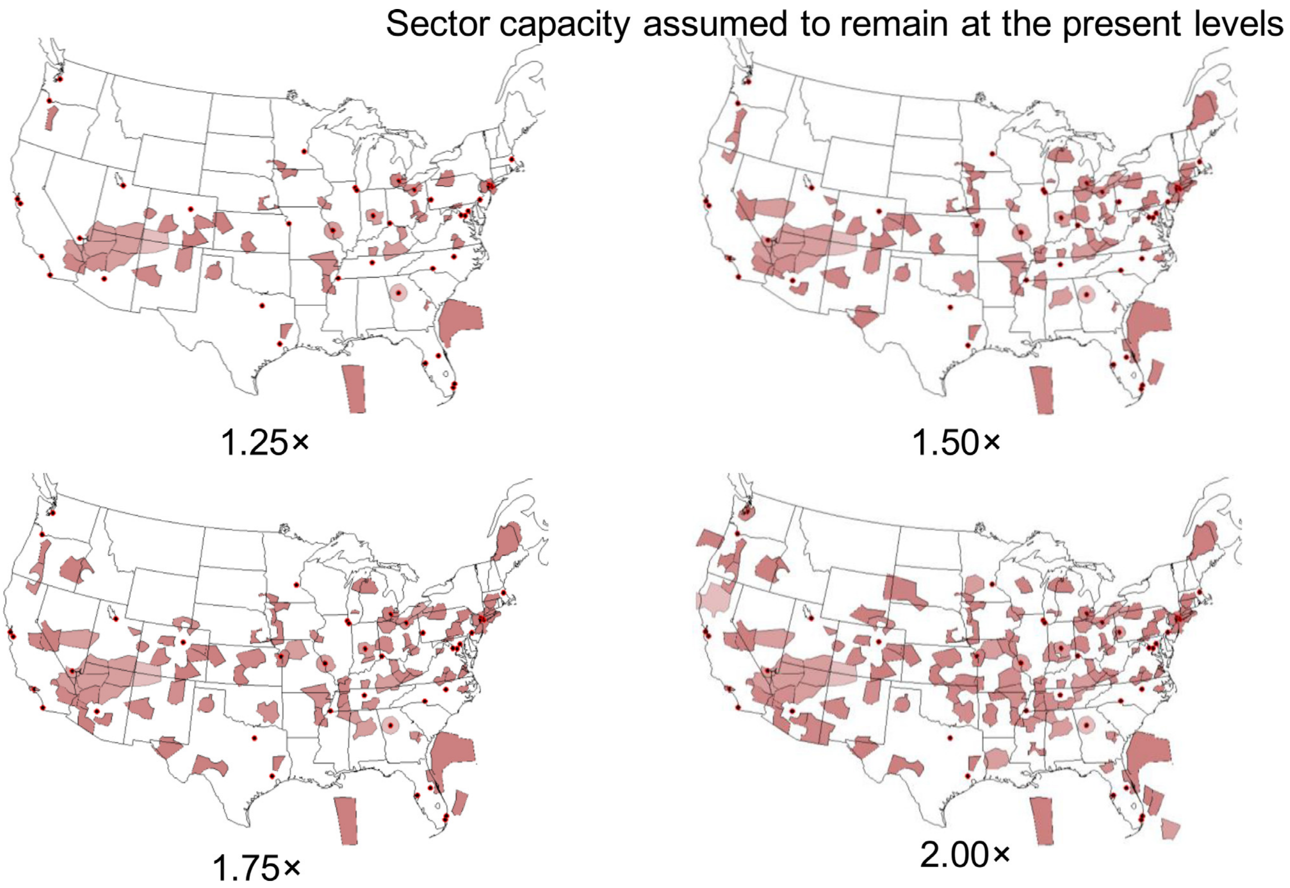

Fig. 16. Analysis of the impact of increasing traffic volume on choke points in the national airspace system.

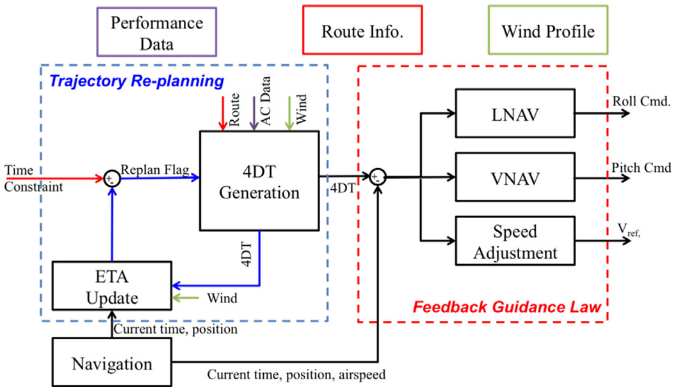

Fig. 17. Schematic of TOAC.

The main control loop in Fig. 17 can be for realizing different for realizing alternate TBO concepts. In the time-based metering concept in which RTA is assigned at some specific waypoints, the main control loop is trajectory re-planning loop. Some of the state-of-the-art FMSs already have this capability, termed as the RTA capability (Jackson & O'Laughlin, 2007; Klooster, Wichman, & Bleeker, 2008). Another operational concept is the 4DFMS concept wherein aircraft is required to track the 4DT continuously after the conclusion of the negotiations between aircraft and ground. In this concept, the feedback guidance loop in Fig. 17 is a main control loop (Garrido-Lopez, D'Alto, & Ledesma, 2009; Vaddi, Sweriduk, Tandale, & Cate, 2012).

The TOA accuracy for various TOAC concepts has been evaluated through Monte-Carlo simulation in the flight arrival phase (Vaddi, Bai, & Park, 2015). Fig. 18 shows sample results of these Monte-Carlo simulations along KSFO MODESTO and KLAX RIIVR standard arrival routes. Each box plot in the figure was obtained from 3000 simulation runs with randomly generated actual wind fields. The TOA error denotes the time difference between RTA and actual TOAs. The results show that not only the accuracy (mean error) but also the uncertainty level (standard deviation) can be substantially improved using advanced TOACs compared to the open-loop LNAV/VNAV guidance.

Interval Management (IM) is another time-based spacing concept that is currently under consideration. In the IM concept, the inter-arrival time between two aircraft is assigned instead of RTA of each aircraft. Several algorithms to achieve IM concept have been proposed in the literature. One of them is the constant time algorithm control which seeks to control crossing times at specific markers relative to target vehicle to ensure the specified interval (Hoffman, Pene, & Zeghal, 2006; Ivanescu, Shaw, Zeghal, & Hoffman, 2007). The Airborne Spacing for Terminal Area (ASTAR) algorithm developed by NASA (Abbott, 2002) controls RTA of ownship at specific waypoint to cross that point at the assigned interval after targetship crossed that waypoint. The ASTAR predicts the targetship trajectory and estimates ETA of targetship at a specific waypoint. Based on this prediction, ASTAR re-plans ownship trajectory to achieve time interval between targetship and ownship. The ASTAR algorithm has been extensively evaluated in Human-In-the-Loop experiments (Barmore, Abbott, & Capron, 2005). Time-To-Go (TTG) algorithm, which is similar to ASTAR (Weitz & Hurtado, 2011, 2012) is an approach in which the targetship trajectory is also required for interval management. Several different IM algorithms have been compared and evaluated (Abbott, 2009; Barmore, Smith, Palmer, & Abbott, 2012).

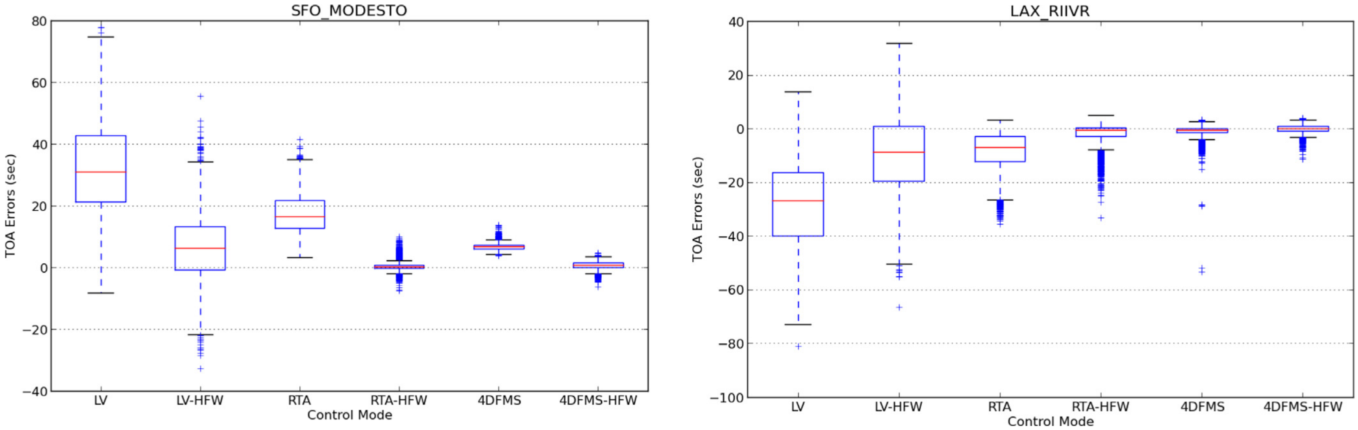

Fig. 18. Statistics of TOA accuracies with different TOAC capabilities, LV – LNAV/VNAV, HFW- high fidelity wind (Vaddi et al., 2015).

TBO concept is rapidly evolving, and may see field deployments over the next decade. Already, portions of this concept are being implemented to create a more resilient air traffic management system.

5. Conclusions and future work

This paper summarized the motivating factors driving the development of modern air traffic management systems, and identified some of the dynamics and control technologies that play a central role in its evolution. Although not an exhaustive survey on the subject, the main purpose of this paper is to pique the interest of dynamics and controls researchers to apply their expertise to advance the next-generation air traffic management technologies.

It is hard to imagine the modern world without aviation. It enables the delivery of personnel and supplies for disaster relief around the world. It may provide better coordination of human and material resources to manage global pandemics. It provides access to widely dispersed markets for perishable goods from all over the world. It enhances opportunities for more frequent cultural interchange, improving the understanding between peoples and nations. As the world operation population increases and the standard of living improves, aviation will become the main mode of human transportation and commerce. It is imperative that this industry continues to grow, to enable and much tighter integration of world economies to ensure continued world prosperity. Air Traffic Management Systems will be a key enabler of this integration.

Acknowledgment

NASA support under multiple contracts for the research discussed in this paper is gratefully acknowledged.

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