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A Study of Near-Optimal Endurance-Maximizing Periodic Cruise Trajectories

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A near-optimal periodic solution to the maximum-endurance cruise problem is investigated. Point-mass models are developed for different types of aircraft. Energy-state methods are used to determine minimum-fuel climb and maximum-endurance descent schedules in the altitude-airspeed plane, which are then pieced together with transition arcs to form a periodic cruise solution. A trajectory tracking controller is designed to make the point-mass models track the periodic cruise trajectories. The tracking controller is designed using the feedback linearization methodology. Closed-loop simulations are then used to compute the fuel consumption resulting from the use of periodic trajectories. These values are compared to the steady-state, optimal-endurance cruise fuel consumption values. For an F/A-18 aircraft model, it was found that savings of about 17% could be realized if the engines can be turned off when the aircraft is not on the climb schedule. However, if the throttle cannot be set below flight idle, the periodic cruise trajectory is found to produce worse performance than the steady-state cruise, primarily due to poor specific fuel consumption at idle throttle setting. Simulations with a model of an F-4 in periodic cruise with idealized engine characteristics non-zero minimum throttle did show a modest improvement over the steady-state cruise performance, but only by 2.7%.

Nomenclature

D	=	drag
E_s	=	specific energy
g	=	acceleration of gravity
h	=	altitude
L	=	lift
pla	=	power lever angle
m	=	mass
T	=	thrust
V	=	velocity
w_f	=	fuel flow rate
α	=	angle of attack
γ	=	flight path angle

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I. Introduction

The cruise segment of a flight forms the dominant portion of most fixed-wing aircraft operations. It has long been known that the steady-state cruise arc can be non-optimal for maximizing range or endurance[1]. Developmental efforts on digital flight management systems (FMS) during the 1970's led to a thorough re-examination of this problem[2,3]. Specifically, in a series of papers, Speyer and others showed that a periodic trajectory can provide better fuel economy when compared with the steady state cruise arc[4-9]. More recent research has examined the cruise problem using numerical techniques[10,11,12]. Most of the research efforts showed that the maximum achievable fuel savings in periodic cruise for the range optimization problem is less than 5% for conventional aircraft configurations[1-10].

However, in References 11 and 12, Sachs and Christodoulou, using a point-mass model and the multiple shooting method, showed that a periodic optimal solution to the endurance problem (maximum time-aloft for a given fuel load) can be substantially higher, of the order of 50%. Although the aircraft configuration used in References 11 and 12 was not identified, it appears that realistic airframe and engine models may have been employed in that work.

A typical endurance-maximizing trajectory starts at the lowest permissible altitude with a fuel-optimal, full-throttle climb. As the aircraft approaches a chosen maximum altitude, the engine power is set to zero or idle and the aircraft transitions onto a maximum-endurance glide path. Maximum power is applied and the aircraft begins its fuel-optimal climb again towards its high-altitude point as soon as it reaches the minimum permissible altitude. Optimal periodic cruise requires the aircraft to repeat this power-on/power-off cycle over and over. When compared with the classical steady-state cruise, the periodic cruise requires the aircraft to continuously accelerate or decelerate throughout its trajectory.

The present research was motivated by the results of References 11 and 12. The objective of this research was to examine the benefits of a periodic cruise trajectory using a near-optimal approach. The endurance problem is of practical importance in surveillance, reconnaissance and data relay missions. Another use of periodic cruise trajectories is in performing fuel-optimal maneuvers to confuse enemy interceptors.

Instead of attacking the high-order two-point boundary-value problem resulting from the application of optimal control theory[13] to the aircraft point-mass model, the present research seeks to construct near-optimal solutions using the energy-state model[14]. In this approach, a fuel-optimal climb schedule is patched together with an endurance-optimal descent schedule in the altitude-airspeed plane. Both these schedules are constructed using the energy-state model. The transition arcs between the climb and descent schedules are constructed as minimum energy loss trajectories. A trajectory-tracking controller is used to make the point-mass model follow the periodic altitude-airspeed schedule.

The periodic cruise analysis was applied to two different aircraft models. The first was the F/A-18 aircraft and the second was an F-4 aircraft incorporating a simpler engine model. For the F/A-18 aircraft it was found that a 17% improvement in aircraft endurance can be obtained if the engine was allowed to be completely shut off during the transition arcs and the descent schedule. However, requiring the aircraft to fly the transition and descent arcs at flight idle throttle setting completely eliminated this saving, primarily due to the poor specific fuel consumption at flight-idle throttle setting. For the F-4 aircraft fuel savings of the order of 3% has been observed even in if the engine is not allowed to be completely shut off.

An additional investigation using the hodograph set was undertaken to examine the potential for fuel savings in employing periodic cruise trajectories. According to optimal control theory, for optimal solutions to exist, the hodograph set or the extended velocity set has to be convex. Classical theory of periodic cruise[4,7,8] asserts that the periodic cruise becomes optimal whenever the hodograph set is nonconvex. It is shown that the hodograph set for the F/A-18 aircraft is nonconvex if the throttle is allowed to transition all the way to zero. If the throttle cannot be moved below flight idle, which is an important practical constraint usually in effect, the hodograph set turns out to be convex.

Section II will discuss the two different aircraft models used in the present study, and the design of trajectory-tracking controllers. Section III will present the methods used in determining the steady state and periodic cruise trajectories, and will present the results of simulations with different aircraft models. Section IV will discuss the hodograph set analysis for periodic cruise problem that helps explain the results of Section III. Section V will present the conclusions from this research and some recommendations for future work.

II. Modeling and Control

This section will discuss the development of the point-mass models for different aircraft, and the aerodynamic and propulsion models used in each. Models for the F/A-18 and the F-4 will be discussed. A nonlinear controller for tracking prescribed trajectories is also developed. This controller is derived using the feedback linearization method, which transforms the nonlinear dynamic model into a linear form. A control is computed for the linear system, and then is transformed back to the original system to give the actual control.

A. Point-Mass Model

The states of the point-mass model of an aircraft in symmetric flight are altitude (h), airspeed (V), and flight path angle (γ). The point-mass equations of motion are:

$$\dot{h} = V \sin\gamma \tag{1}$$

$$\dot{V} = \frac{T\cos\alpha - D}{m} - g\sin\gamma \tag{2}$$

$$\dot{\gamma} = \frac{g}{V}\left(\frac{L + T\sin\alpha}{mg} - \cos\gamma\right) \tag{3}$$

$$\dot{m} = -w_f \tag{4}$$

where T is thrust, L is lift, D is drag, m is mass, and g is the acceleration of gravity. The control variables are the throttle, which affects T, and angle of attack (α), which affects L and D. Note that in some studies[10, 11] the lift coefficient is the control variable, and the drag coefficient is expressed as a function of the lift coefficient, and zero-lift drag coefficient. Angle of attack is assumed to be small, such that the component of thrust along the lift direction is ignored. The fuel flow rate w_f and thrust T are generally functions of altitude, Mach number and throttle setting. An important assumption included in previous research[1 – 12] is that the fuel flow rate and thrust depend linearly on thrust.

Note that the mass is held constant, although in the simulations to be discussed later, the fuel flow rate is integrated with time to determine the amount of fuel consumed. It is standard assumption that the amount of fuel consumed over the period in question is a small enough fraction of the total aircraft weight that the difference is negligible.

Although the use of the point mass models produce higher-fidelity results, a lower-order model, namely the energy state model is often used[14] in trajectory optimization studies to obtain insights about the solutions. Due to the preliminary nature of the present research, the energy state model will be used to assemble periodic cruise trajectories in the present research. Further details of this modeling will be given in Section III.

B. F/A-18 Model

A point-mass model was developed using code from NASA Dryden for the F/A-18 HARV. The right hand sides of the point mass model (i.e. forces) are computed using two functions. The aerodynamics function returns the lift and drag coefficients for the given Mach number, altitude, angle of attack and angle of sideslip. The propulsion function returns net thrust and fuel flow rate at the given Mach number, altitude, angle of attack, and power lever angle (*pla*); i.e. throttle setting. The mass was assumed to 1087.8 slugs (35,000 lbm) and the reference area was 400 sq. ft.

C. F-4 Model

A model of the McDonnell-Douglas F-4 Phantom II was used in other studies of the range-optimal cruise problem[10,17]. In this model, the throttle (η) is expressed as a percentage of maximum, and the lift coefficient is the second control variable. The drag coefficient is defined as a function of the lift coefficient and the zero-lift drag coefficient. The aircraft mass was 1164.9 slugs, and the reference area was 530.11 sq. ft. It was not specified what the limits of the propulsion model were, but a comparison of the propulsion model with other sources suggested that the maximum throttle corresponds to military power[18,19]. The maximum value of the lift coefficient was not specified, so a value of 1.0 was assumed. Using this value, the level flight envelope was computed and is shown in Figure 2. The slightly supersonic maximum speed indicated in Figure 1 is therefore somewhat unrealistic, but otherwise the envelope is qualitatively similar to the F/A-18 flight envelope.

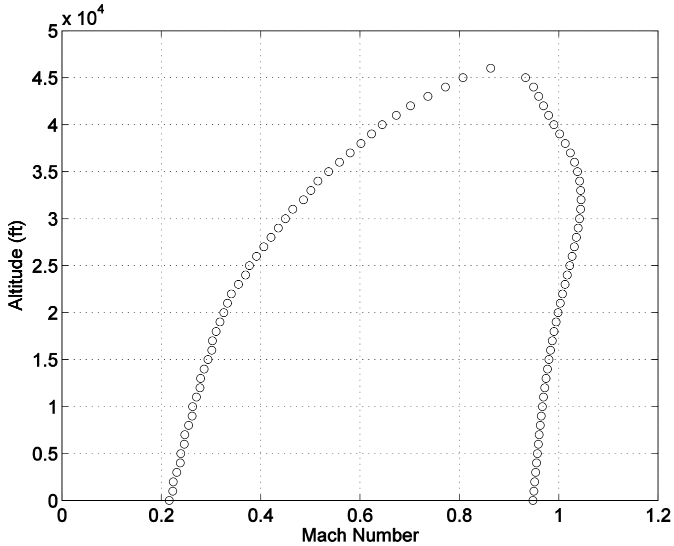

Figure 1. Level Flight Envelope for F-4

It should be noted that in this model the specific fuel consumption at a given altitude and airspeed is independent of the throttle setting. Typically the thrust specific fuel consumption of a gas turbine increases with decreasing throttle[20], so this model is somewhat inaccurate.

D. Trajectory-Tracking Controller

As will be shown in Section III, reduced-order modeling generates periodic cruise trajectories in the altitude-airspeed plane. Thus, a feedback controller is needed to make the point-mass model to follow a prescribed trajectory. In this formalism, the trajectories to be tracked are not expressed as explicit functions of time. Instead, as will be discussed in the following section, the optimal climb and descent profiles are specified as points on the h-V plane at fixed energy levels. A controller was designed using feedback linearization[21] to track a reference altitude command, with the angle of attack as the control variable. Since the throttle is set at fixed values along segments of the periodic cruise trajectory, and the speed V is controlled by changing the flight path angle γ, which is controlled by changing the lift force L or the angle of attack α. The controller is derived as follows: differentiating Equation 1 with respect to time gives

$$\ddot{h} = \frac{\partial}{\partial h}(V \sin\gamma)\frac{dh}{dt} + \frac{\partial}{\partial V}(V \sin\gamma)\frac{dV}{dt} + \frac{\partial}{\partial \gamma}(V \sin\gamma)\frac{d\gamma}{dt} \tag{9}$$

Taking the partial derivatives and substituting from Equations (1)-(3),

$$\ddot{h} = 0 + \sin\gamma\left(\frac{T\cos\alpha - D}{m} - g\sin\gamma\right) - V\cos\gamma\,\frac{g}{V}\left(\frac{L + T\sin\alpha}{mg} - \cos\gamma\right)$$

$$= \sin\gamma\left(\frac{T\cos\alpha - D}{m}\right) - g + \cos\gamma\left(\frac{L + T\sin\alpha}{m}\right) \tag{10}$$

The previous equation is written as:

$$\ddot{h} = F + G\nu \tag{11}$$

where F, G, and ν are defined by

$$F = \sin\gamma\left(\frac{T\cos\alpha - D}{m}\right) - g \tag{12}$$

$$G = \cos\gamma \tag{13}$$

$$\nu = \frac{L + T\sin\alpha}{m} \tag{14}$$

Now let $F + G\nu = \mu$, where μ is defined by a PID feedback control law:

$$\mu = K_P\left(h_c - h\right) + K_I\int_o^{tf}\left(h_c - h\right)dt + K_D\left(\dot{h}_c - \dot{h}\right) \tag{15}$$

where h_c and $\dot{h}_c$ are determined from the periodic cruise schedule and the gains K_P, K_I, K_D are chosen to give good trajectory tracking. Once the control for μ is computed, the pseudo-control ν can be solved for in terms of F and G. Note that F and G functions can be computed using the states, thrust and aerodynamic models. Since $\gamma < \pi/2$, G^{-1} always exists. The required value of the actual control variable α is then found by iteratively solving Equation (14). A controller can also be derived to follow a velocity command, although this requires partial derivatives that must be numerically approximated.

III. Optimal Cruise

In this section, the aircraft models and trajectory controllers described in Section II are used to assemble periodic cruise trajectories and to assess the potential fuel savings in employing periodic cruise. The following sections will first discuss the derivation and computation of steady-state and periodic cruise trajectories. The results of simulations with various aircraft types flying periodic cruise trajectories are then presented.

A. Steady-State Cruise

Aircraft normally spend most of a flight at or near some fixed altitude and airspeed. Assuming a steady-state condition, the airspeed and altitude can be optimized for either maximum range or maximum endurance. The steady-state cruise condition therefore serves as the basis for comparison with periodic trajectories.

The maximum-endurance cruise problem is to find the level-flight condition that maximizes the time aloft for a given amount of fuel consumed, or equivalently, minimizes the fuel consumption rate, which is a function of airspeed, altitude, and throttle. The maximum-endurance cruise point was found using the MATLAB® Optimization Toolbox function "fmincon"[16]. The free parameters were h, V, α, and pla, the cost function was the fuel flow rate w_f, and equality constraints were imposed on the sum of the horizontal forces and the sum of the vertical forces; i.e. the trim condition.

B. Periodic Cruise

As discussed in Section I, previous research[4-12] has demonstrated that a periodic cruise trajectory can provide better cruise performance than the steady-state condition. Although numerical methods for trajectory optimization can be used for these studies as in References 6, 10 - 12, the present research will employ energy state methods to construct near-optimal periodic cruise trajectories. The central idea is to patch together fuel-optimal climb path with a endurance-optimal glide trajectory to form the periodic cruise trajectory. Both these arcs can be determined using energy state methods [14].

The energy-state model is a lower-order approximation to the point-mass model in which the states are mass m and specific energy:

$$E_s = h + V^2/2g \tag{16}$$

The dynamic equations are then:

$$\dot{E}_s = \frac{V(T\cos\alpha - D)}{mg} \tag{17}$$

$$\dot{m} = -w_f \tag{18}$$

The altitude or the airspeed are treated as the control variable in this model. Angle of attack α is often assumed to be small for these analyses. However, in the present research, α is found by satisfying the conditions for vertical equilibrium.

1. Energy State Method for Minimum-Fuel Climb

It has been shown in References 14 and 15 that the fuel-optimal climb schedule can be determined using the necessary condition:

$$\left.\frac{\partial}{\partial h}\frac{V(T-D)}{\dot{m}}\right|_{E=Const} = 0, \quad T \geq D\,, \tag{19}$$

At each energy level, this expression specifies an optimal altitude. By satisfying the above necessary condition at various energy levels within the aircraft flight envelope, a fuel-optimal climb schedule can be constructed in the altitude-airspeed plane.

The minimum-fuel climb schedule was determined by first selecting a set of energy levels covering the flight envelope. For each specific energy level E_s, with *pla* fixed at the maximum military power, the Optimization Toolbox[16] function "fmincon" was used to find V, α, and γ that maximize the function:

$$f = \frac{V(T\cos\alpha - D)}{\dot{m}} \tag{20}$$

with inequality constraints on the sum of the horizontal forces, the sum of the vertical forces, and the flight path angle:

$$\sum F_x \geq 0, \quad \sum F_z \geq 0, \quad \gamma \geq 0\,, \tag{21}$$

and altitude is constrained by the energy equation E_s.

2. Energy State Method for Maximum-Endurance Descent

In a similar manner, the power-off glide trajectory that maximizes the aircraft endurance can be determined from the energy-state model by satisfying the necessary condition[14]:

$$\left.\frac{\partial[V(T-D)]}{\partial h}\right|_{E=Const} = 0, \quad T \leq D \tag{22}$$

As in the computation of the fuel-optimal climb path, this necessary condition can be satisfied at various energy levels within the flight envelope to determine an endurance maximizing glide schedule in the altitude-airspeed plane.

The maximum endurance descent was computed for two cases: with the throttle at flight idle and with the throttle at 0. In most works on this subject, it is assumed that thrust and fuel flow are zero. However, turning off the engines in flight may not be acceptable in many cases, especially if the aircraft does not carry an auxiliary power unit. To compute the schedule, a set of energy levels was chosen. For each specific energy level E_s, the Optimization Toolbox function "fmincon" was used to find V, α, and γ that minimize the energy loss rate:

$$f = \frac{V\left(D - T\cos\alpha\right)}{mg} \tag{23}$$

(the numerator is assumed positive) with an equality constraint on the sum of the vertical forces,

$$\sum F_z = 0\,, \tag{24}$$

an inequality constraint on the flight path angle

$$\gamma \leq 0, \tag{25}$$

and altitude is constrained by the energy equation:

$$E_s = h + V^2/2g. \tag{26}$$

C. Periodic Cruise Results, F/A-18

Simulations were performed with military power during the climb phase, and then the throttle was set to either flight idle or zero. The power was reduced when the specified maximum energy level was reached, and a positive flight path angle was maintained until the speed decreased to the point where the descent schedule was reached. The best angle for this arc was found to be around 11.5 deg. The descent schedule was then followed down to minimum altitude. Constant-energy transfer arcs between the climb and descent paths (with intermediate throttle) were found to lower the efficiency.

1. Periodic Cruise with Zero Minimum Throttle

The trajectory for the zero-power descent is shown in Figure 2. The values have been normalized. Iterations on the value of maximum specific energy were performed to find the value that gave the most savings. The average fuel consumption rate came out to be a 15.5% improvement over the steady-state cruise condition. Note that the amount of fuel consumed was only a few percent of the assumed aircraft mass during each cycle, so the assumption of constant mass for the analysis seems reasonable. It should be noted that the model does not include windmill drag or inlet drag when the throttle is set to zero, so the true savings should be expected to be less than this figure.

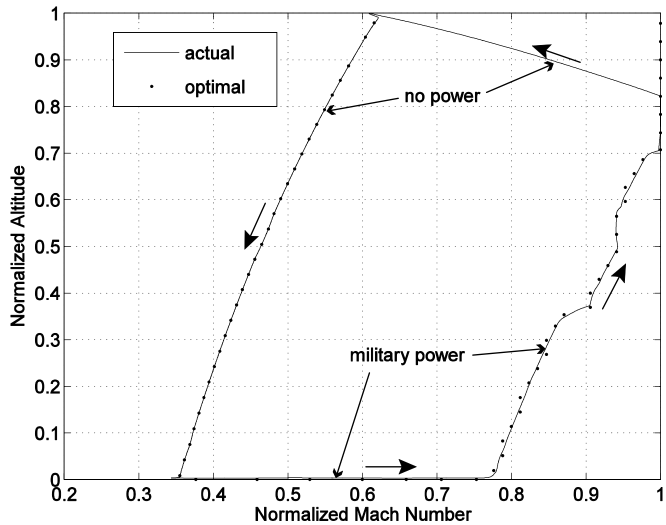

Figure 2. Trajectory #1: Optimal Periodic Cruise with Zero Minimum Throttle

2. Periodic Cruise with Idle Minimum Throttle

The normalized results for the idle power case are shown in Figure 3. The same maximum specific energy was used to switch the power setting. The average fuel consumption rate was 27.4% loss of efficiency relative to the steady-state cruise point. The time histories are very similar to those of the previous case, despite the fact that the engines are on throughout the flight. Lowering the maximum specific energy level results in a greater loss of efficiency if the transfer arc is flown at minimum thrust and constant γ. Also, there is a slight loss of efficiency if the transfer arc is flown at constant energy.

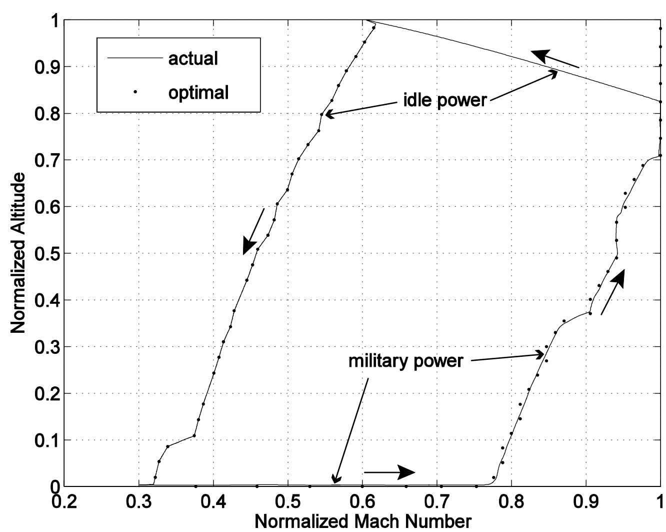

Figure 3. Trajectory #2: Optimal Periodic Cruise, Flight-Idle Minimum Throttle

3. Improved Transition Arcs

By making some modifications to the transition trajectories between fuel optimal climb and endurance-maximizing descent arcs, some additional fuel savings were realized for the zero minimum throttle case. The same maximum specific energy was used. In this case the transition between the climb and descent schedules was not at a constant flight path angle, and there is a dive at the bottom of the descent to trade some altitude for airspeed. The controller was altered to follow altitude rate commands during the transfer arcs because this provided better trajectory tracking.

The average fuel consumption rate showed a 16.7% improvement over the steady-state cruise condition. The normalized results of the simulation are shown in Figure 4. The dive at the end of the descent can be seen in Figure 4. A higher flight path angle is reached in the high altitude transfer.

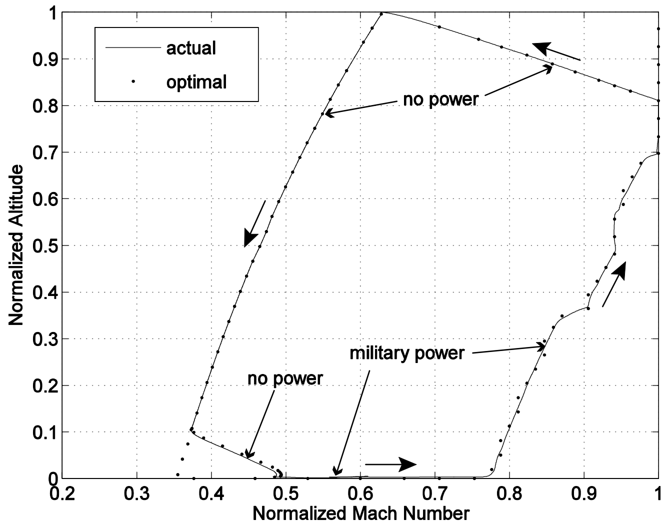

Figure 4. Trajectory #3: Optimal Periodic Cruise with Modified Transfer Arcs

D. Periodic Cruise Results for the F-4 Aircraft Model

The steady-state cruise condition for this model was found to be: h = 17592.7 ft., V = 522.9 ft/s, η = 0.3339. The fuel consumption rate for this condition is 3792 lbm/hr. The engine model is linear in throttle and it was difficult to determine the throttle setting for flight idle. It was decided to use a value of 0.1 (10%) for the minimum throttle, which should be a conservative estimate. Several periodic trajectories were constructed using different values for maximum energy. Fuel savings over steady-state cruise were achieved using a periodic trajectory, but the best that could be achieved, by varying the maximum energy, was 2.7% (time of flight = 875.3 s, fuel consumed = 897.2 lbm, average fuel consumption rate = 3690 lbm/hr). The results are shown in Figure 5. The maximum altitude was approximately 24,000 ft, and the minimum altitude was 100 ft.

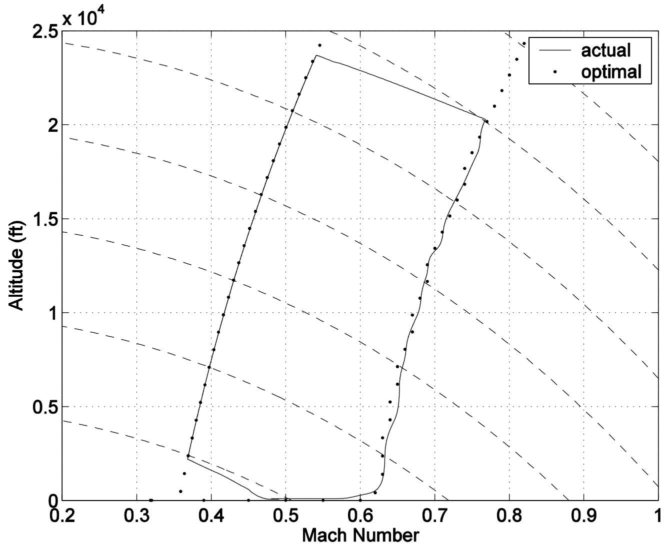

Figure 5. Periodic Trajectory for F-4 Model

E. Hodograph Analysis

In this section, analysis of hodograph sets is used to verify the results of the previous simulations. A hodograph set consists of the set of state rates that can be achieved using admissible controls[13]. For the aircraft endurance study, the hodograph set is two-dimensional with coordinates being the rate of change in specific energy, $\dot{E}_S$, and fuel flow rate, $\dot{m}$ (often represented by w_f). The energy-state model is an approximation to the point-mass model, where energy and mass become the state variables. The hodograph is formed by determining the boundary of possible rates of the state variables E_s and m that can be achieved for admissible values of the control variables. Since the steady-state cruise is performed at constant energy, the corresponding fuel flow rate is given by the value of the fuel flow rate at the point where the hodograph set boundary crosses the zero-energy-rate axis ($\dot{E}_S = 0$, or E_s = constant, implies no acceleration; i.e. the aircraft is in steady state). Optimal control theory states that optimal solutions exist only if the hodograph set is convex[13]. If the hodograph set is nonconvex, so that a straight line that is tangent to two points on the hodograph set crosses the zero energy rate axis at a lower value of the fuel flow rate than the hodograph set itself, then the existence of a switching control with better cruise performance is indicated [6-8].

Since the control variables in the energy state model are altitude and throttle setting, the hodograph sets in the following sections were constructed by selecting a range of altitude points around the cruise point, and a range of throttle points. Airspeed was determined from the energy equation, where the energy is fixed at the cruise point. Every point for which the aircraft can be trimmed in the vertical axis is an admissible point. Enough points were generated to get an idea of where the boundaries of the set were, and the MATLAB® function "convhull" was used to compute the convex hull of the set, which is the smallest convex set that contains all of the points in the original set. If the hodograph is nonconvex, then the convex hull will form the line tangent to the hodograph as described in the previous paragraph.

1. Analysis of the F/A-18 Aircraft Model

Hodograph sets for minimum throttle settings of 0 and flight idle are shown in Figure 6 and Figure 7, respectively, where the fuel flow has been normalized to the steady-state cruise value. The dashed line is the convex hull of the set. The two graphs are the same except for the points where fuel flow is zero, and the convex hull thus differs. In the first graph the convex hull can be seen above the boundary of the set in a region around the cruise point ($wf_{norm} = 1, \dot{E}_S = 0$). In the second graph the convex hull is indistinguishable because the set is convex, meaning no savings are possible for this aircraft if the minimum throttle setting permitted is flight idle.

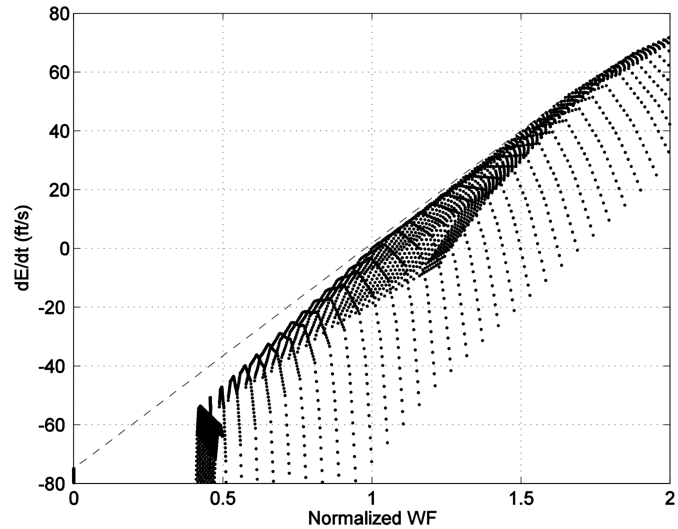

Figure 6. F/A-18 Hodograph Set for Zero Minimum Throttle

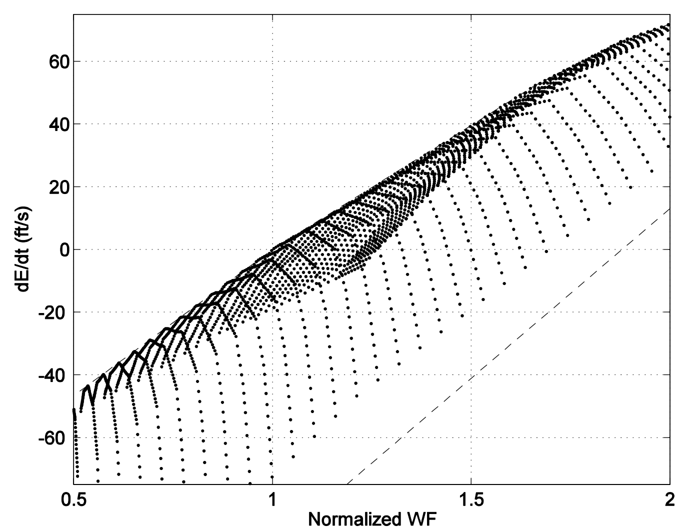

Figure 7. F/A-18 Hodograph Set for Idle Minimum Throttle

2. Hodograph Set Analysis for the F-4 Aircraft

A hodograph set was constructed for the F-4 aircraft using 10% as the minimum value of the throttle and is shown in Figure 8. The hodograph set shows nonconvexity, indicating that fuel savings are possible. Lower minimum throttle setting may permit better fuel savings.

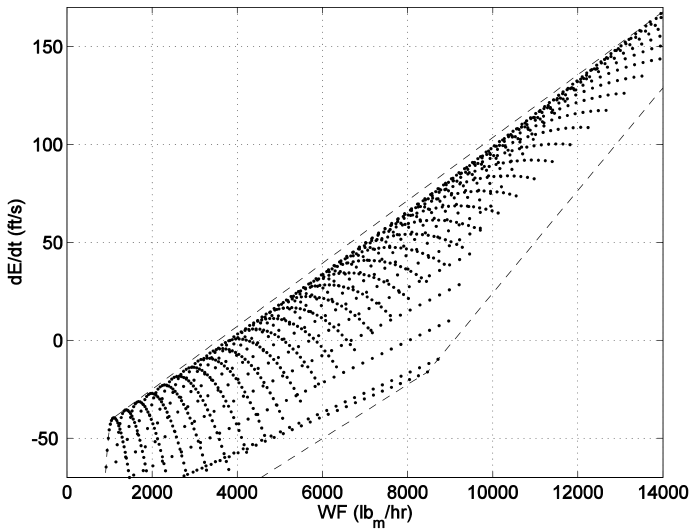

Figure 8. Hodograph Set for F-4 Model with 10% Minimum Throttle

IV. Conclusions

This paper presented a near-optimal study of the aircraft endurance problem using the energy-state approximation. Periodic cruise trajectories were constructed by patching together a fuel-optimal climb schedule with a maximum-endurance glide path, with minimum-energy-loss transition arcs. These schedules in the altitude-airspeed plane were then used as the tracking command to a point-mass model with an altitude tracking control law. Two aircraft models were used in the study.

The periodic trajectory produced fuel savings of the order of 17% for the F/A-18 model if the minimum permissible throttle setting is assumed to be zero. However, if the minimum throttle setting was constrained to be the "flight idle" setting, which is the case in normal flight operations, the fuel savings in periodic cruise disappeared completely. This is mainly due to the fact that in the latter case, fuel continues to be consumed although little or no thrust is being generated. Moreover, the specific fuel consumption at idle is significantly degraded when compared with the maximum power setting. Turning off the engines in flight is undesirable because the aircraft's altitude and airspeed are close to minimum when the engines need to be restarted, so there is little safety margin if the engines do not restart immediately. In addition, turning off the engines immediately after running at full power for several minutes may produce undesirable thermal effects.

The F-4 model did exhibit small savings in endurance even with nonzero minimum permissible throttle setting. It is important to recall from Section II that a linear thrust-throttle relationship is employed in this model. Prior research by other investigators using this model have achieved a small reduction in fuel consumption in periodic cruise for the *range* problem, and some researchers have postulated that an aircraft which achieves savings in range would achieve substantially more savings for the *endurance* problem.

Since the present research has demonstrated the crucial role played by the engine model in the outcome of periodic cruise, it is recommended that future research should conducted using accurate thrust and fuel flow rate models.

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